Capacity Investment Timing by Start-ups and Established Firms in New Markets

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We analyze the competitive capacity investment timing decisions of both established firms and start-ups entering new markets, which have a high degree of demand uncertainty. Firms may invest in capacity early (when uncertainty is high) or late (when uncertainty has been resolved), possibly at different costs. Established firms choose an investment timing and capacity level to maximize expected profits, whereas start-ups make those choices to maximize the probability of survival. When a start-up competes against an established firm, we find that when demand uncertainty is high and costs do not decline too severely over time, the start-up takes a leadership role and invests first in capacity, whereas the established firm follows; by contrast, when two established firms compete in an otherwise identical game, both firms invest late. We conclude that the threat of firm failure significantly impacts the dynamics of competition involving start-ups.

1. Introduction

Firms entering new markets face numerous operational challenges. Among the most crucial are issues related to capacity investment. Particularly when the size of a market is uncertain, two common yet difficult decisions are how much capacity to invest in and when to do it. When choosing how much capacity to build or reserve with a supplier, the trade-off is clear: too much capacity results in underutilized facilities (if output is reduced to match market demand) or depressed prices (if output remains high despite low demand), whereas too little capacity results in reduced sales and suboptimal profit and growth.

Timing the capacity investment decision presents even subtler considerations. Uncertainty surrounding market size typically reduces over time, meaning a firm that invests in capacity early is subject to a higher degree of demand uncertainty than a firm that postpones the investment decision. On the other hand, in competitive situations, a firm investing earlier than its rivals becomes the first mover in the market, which may yield a strategic advantage. Indeed, the cost of capacity itself may change over time, either increasing (e.g., if contract capacity becomes scarce as the market matures) or decreasing (e.g., if learning enables lower-cost processes). These factors combine to make the decision of *when* to invest in capacity just as difficult and perilous, if not more so, as the decision of *how much* capacity to build or purchase.

The timing of capacity investment when entering new markets is precisely the issue that we consider. We first examine stylized monopoly models in which the sole entrant to a new market must build or source capacity in anticipation of future demand. Eventual market size is uncertain, and the firm is allowed to invest in one of two periods: if the firm invests *early*, then it makes the capacity decision before knowing market size, whereas if it invests *late*, all demand uncertainty is eliminated and capacity is built or sourced after learning market size. The cost of capacity is allowed to vary between periods. Thus, a monopolist firm must trade off the value of information (which is gained if the investment decision is delayed) with potential cost advantages from early investment.

Because new markets are often pursued by nascent firms, we focus on how the timing of capacity investment differs between *start-ups* and *established firms*. We consider the primary difference between these two types of firms to be the threat of bankruptcy or firm failure. Large established firms diversifying into new markets are unlikely to face imminent peril should

demand in that market turn out to be low; start-ups, on the other hand, are typically smaller firms wholly invested in a single market, and thus, to a far greater extent than their established counterparts, face potentially disastrous consequences should the market fail to materialize as expected. The presence of this risk, combined with the high degree of demand uncertainty that typically accompanies the development of a new market, implies that start-ups should have a utility function that takes into account the risk and consequences of failure. Hence, in our model, the objective of a start-up is to time the capacity investment decision to maximize the *probability of survival*. Established firms, by contrast, do not face an imminent risk of failure, and hence make capacity decisions to maximize *expected profit*.

In the monopoly setting, we examine how start-ups differ in their capacity timing decisions from established firms, characterizing how market uncertainty, capacity costs, and the threat of failure influence both capacity levels and investment timing. We find that established firms are likely to prefer late investment even if early investment is cheaper, because the flexibility to respond to market conditions engendered by late investment allows the firm to capture higher profits, particularly in high-demand states. By contrast, start-ups prefer to invest in capacity whenever capacity is least expensive—that is, if capacity costs increase over time, start-ups prefer early investment—because lower capacity costs minimize the threshold market size that results in firm survival and hence maximize the probability of survival.

We then proceed to analyze duopoly models in which two firms simultaneously consider entry into a new market. In addition to all of the trade-offs inherent in the monopoly model, the competitive interaction introduces a strategic aspect to the capacity investment timing decision: a firm investing earlier than a competitor may gain a leadership position in a sequential game. We find that when a start-up competes with an established firm, if market uncertainty is high (as in a new market) and costs do not decline severely over time, then the unique equilibrium is for the start-up to invest early, whereas the established firm invests late. By contrast, when two established firms compete, the only equilibrium when demand uncertainty is high is simultaneous: both firms invest late. We thus conclude that the threat of failure experienced by a start-up tends to push capacity investment earlier—in both monopolistic and competitive situations—and leads to asymmetric investment timing equilibria in which start-up firms, remarkably, act as first movers in new markets, despite the apparent advantages of established firms in terms of resources and technology.

In this regard, our findings relate to several streams of research, for example, the literature on disruptive innovation. The seminal works on this topic are Christensen and Bower (1996) and Christensen (1997); Schmidt and Druehl (2008) provide a recent review. A *disruptive innovation* is an improvement in a product or service that fundamentally changes its cost, performance, or target market in new or unexpected ways.[1] Such innovations are typically enabled by scientific, technological, or process advancements; for example, the rise of inexpensive, physically compact desktop computers enabled the emergence of the personal computing market over the minicomputer and mainframe markets, and the development of cheap, tiny, digital flash storage technologies helped contribute to the dominance of digital photography over film photography. A recurring question in this literature is, why do large, established firms typically fail to embrace disruptive innovations early, whereas smaller start-up firms often take a leadership role in bringing the innovations to market? Our model supports one possible answer to this question, namely, that it is the natural equilibrium of an endogenous timing game between a start-up and an established firm.

2. Related Literature

There are three primary streams of research related to our work: the operations literature on capacity investment under uncertainty, the economics literature on competitive capacity investment, and the strategic management literature on new market entry and disruptive innovation. The latter topic was discussed in §1; here, we briefly review the remaining two broad areas, with further references to relevant works included throughout the remainder of this paper.

Our model is one of capacity investment with stochastic demand. As such, it is related to the extensive operations literature on this topic; see the comprehensive review by Van Mieghem (2003). A number of papers consider the value of delaying capacity investment to obtain more accurate demand information; see, e.g., the literature on postponement, such as Van Mieghem and Dada (1999), Anand and Girotra (2007), and Anupindi and Jiang (2008), though these works differ from ours in that they do not consider the possibility of firm bankruptcy and the implications it may have on the timing incentives of a

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[1] We abstract from the details of innovation and focus on the outcome of innovation resulting in highly uncertain new markets; thus, although we use the term "disruptive" because it invokes an image of significant market upheaval and uncertainty, innovation in our context could in fact be any of the four types of technological change described by Lange et al. (2009)—sustaining, disruptive, architectual, and competence destroying discontinuities—as long as the result is uncertainty in the size of the resultant market.

start-up firm. Some works of particular relevance in this stream include Archibald et al. (2002), Babich et al. (2007), Babich (2010), Swinney and Netessine (2009), and Boyabatli and Toktay (2007), all of which consider the impact of bankruptcy risk on capacity or inventory decisions. Tanrisever et al. (2009) consider the related issue of simultaneous investment into capacity and process improvement in the presence of bankruptcy. Although these papers address various consequences of bankruptcy on operational decisions (including process development, capacity levels, financial subsidies to suppliers, and contracting and sourcing strategies), no paper in the literature, to our knowledge, considers the impact of bankruptcy or firm failure on capacity investment *timing*. Indeed, there is a relative lack of research in the operations literature on the topic of capacity investment timing for entry into new markets.

We analyze duopoly models consisting of two firms strategically investing in capacity before either begins to sell in the market. Similar models, frequently referred to as "endogenous leadership games" in the economics literature, have been studied by Gal-Or (1985), Saloner (1987), Hamilton and Slutsky (1990), Maggi (1996), and Bhaskaran and Ramachandran (2007). Maggi (1996) considers an endogenous leadership game with demand uncertainty, much like ours, although two key differences are that the differing objectives of start-ups (and hence the impact of bankruptcy) are not considered, and further capacity investment may occur in multiple periods (whereas in our model, capacity investment occurs in at most one period, because of, e.g., high fixed costs). Also along these lines is the long stream of research on capacity investment for entry deterrence, pioneered by Spence (1977). In addition, the literature on mixed oligopolies—markets in which competing firms have heterogeneous objective functions—is also relevant. Many of these papers focus on markets in which some firms are private (meaning they maximize profit) and some are public (meaning they maximize social welfare). De Fraja and Delbono (1990) provide a review, and more recent examples of mixed oligopoly models include De Donder and Roemer (2009), who study competition between firms that maximize profit and revenue, respectively, and Casadesus-Masanell and Ghemawat (2006), who analyze a profit-maximizing firm competing against an "open source" firm that prices at marginal cost. To the best of our knowledge, none of these papers analyze a mixed duopoly consisting of a profit-maximizing firm and a bankruptcy-prone firm, nor do they consider the capacity investment timing issue.

Last, there is an extensive literature on entry timing for reasons not related to strategic capacity investment. Some examples include social influence (Joshi

et al. 2009), quality or cost improvements (Lilien and Yoon 1990), product technology (Bayus and Agarwal 2007), and product design (Klastorin and Tsai 2004). Our paper differs from these by focusing solely on the impact of bankruptcy risk on capacity investment timing under demand uncertainty and exploring how such risk impacts timing in duopolistic settings.

3. Monopolistic Firms
In this section, we introduce and analyze two different monopoly models of capacity investment timing in a new market with uncertain demand: §3.1 discusses an established, profit-maximizing firm, whereas §3.2 considers a start-up prone to bankruptcy. We defer all discussion of competition until §4.

3.1. A Monopolistic Established Firm
An established firm (denoted by the subscript e) sells a single product.[2] The quantity of the product released to the market is Q_e. The market price is given by the linear demand curve $p(Q_e) = A - Q_e$. Prior to determining the production quantity, the firm must invest in production capacity K_e, which determines its maximum output. This capacity may be internal to the firm (e.g., if the firm in question is a manufacturer) or external (e.g., if the firm outsources production to a contract manufacturer). There is no constraint on the total amount of capacity that can be built or reserved in either case.

Capacity investment may occur at one of two times: either *early* or *late*. Early investment is sufficiently far in advance of the selling season that the total market size is uncertain. The uncertainty in market size is reflected in the demand intercept, A, which is modeled as a continuous random variable with positive support, distribution function F, mean μ, and variance σ^2.

Late investment, on the other hand, is sufficiently close to the start of the selling season that all uncertainty in A is eliminated; hence, capacity investment is made after observing the realized value of A. Demand uncertainty may be reduced or eliminated via a variety of mechanisms. For example, uncertainty may be resolved exogenously if demand depends highly on overall market or economic conditions at the time of product release, or if demand is a function of overriding consumer trends in the category. The firm may take actions to resolve demand uncertainty, such

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[2] We implicitly assume that the established firm—diversifying into the new market—has already evaluated the impact (if any) that market entry will have on sales of its existing products and determined that entry is profitable; Druehl and Schmidt (2008) analyze this related problem of how new market entry can encroach on sales of existing (substitutable) products.

as performing extensive market research, employing consumer focus groups, or working with retailers to improve forecasts. Last, the firm may even produce some (economically insignificant) number of units (e.g., using outsourced capacity) to sell in test markets, postponing full capacity investment until a later date.

Regardless of when the firm chooses the capacity, the production *quantity* (Q_e) is determined after A has been observed and K_e has been fixed (i.e., just before the selling season), and hence output is subject to the constraint $Q_e \leq K_e$. We assume that capacity investment, whenever it is made, is irreversible. Furthermore, capacity investment can occur in at most one period.[3] The total capacity cost is linear in the amount of capacity reserved, and the marginal cost of capacity may vary over time. The unit cost in the early period is denoted by c_1, and the unit cost in the late period is denoted by c_2. We make no ex ante assumption on the ordering of c_1 and c_2. Costs that decrease over time (i.e., $c_1 > c_2$) may be reflective of exogenous technological or process cost improvements, innovation, or raw material cost decreases; similarly, costs that increase over time ($c_1 < c_2$) could occur if contract manufacturers offer a discount for early investment, if capacity in the later period is scarce, or if second-period capacity must be installed more quickly, incurring expedited construction or configuration costs. The reasons behind intertemporal cost variation are outside the scope of this paper; rather, we will present results that hold conditional on a particular cost trend.

The marginal production cost is zero, and for analytical tractability, we assume that the firm adheres to a production clearance strategy, that is, the firm always produces up to its capacity and releases the maximum quantity to the market, $Q_e = K_e$.[4] The established firm, being a large, diversified company, faces minimal risk of bankruptcy as a result of entry into this new market; hence, facing uncertainty in market size (A), the established firm seeks to maximize *expected profit*, which is denoted by $\mathbb{E}(\pi_e(K_e))$, where the absence of the expectation operator, $\pi_e(K_e)$, is used to denote profit for a particular realization of A. Throughout the analysis, optimal values (capacities,

profits, etc.) are denoted by the superscript $*$. Given this formulation, the firm's optimal expected profit from early capacity investment is

$$\mathbb{E}(\pi_e^*) = \max_{K_e \geq 0} \mathbb{E}((A - K_e - c_1)K_e), \quad (1)$$

whereas the firm's optimal expected profit from late capacity investment is

$$\mathbb{E}(\pi_e^*) = \mathbb{E}\left(\max_{K_e \geq 0}((A - K_e - c_2)K_e) \right). \quad (2)$$

Thus, when the firm is deciding whether or not to invest in capacity in the early period, it must compare (1) with (2). The following theorem provides the details of the optimal capacity timing and investment level.

THEOREM 1. *A monopolist established firm prefers early investment if and only if* $\sigma^2 < (\mu - c_1)^2 - (\mu - c_2)^2$, *yielding optimal capacity* $K_e^* = (\mu - c_1)/2$ *and expected profit* $\mathbb{E}(\pi_e^*) = (\mu - c_1)^2/4$. *Otherwise, the firm prefers late investment, yielding optimal capacity* $K_e^* = (A - c_2)/2$ *and expected profit* $\mathbb{E}(\pi_e^*) = (\mu - c_2)^2/4 + \sigma^2/4$.

PROOF. All proofs appear in the appendix. □

As Theorem 1 demonstrates, an established monopolist prefers early investment if and only if demand uncertainty is low and early investment is cheaper than late investment ($c_1 < c_2$). Note that if capacity costs decrease over time ($c_1 > c_2$), the firm prefers late investment for any feasible variance (i.e., for any $\sigma^2 \geq 0$). If capacity costs increase over time ($c_1 < c_2$), the firm may prefer early or late investment, depending on the level of demand uncertainty.

3.2. A Monopolistic Start-up
A common feature of new markets, particularly those enabled by ground-breaking or unforeseen technological innovation, is that they are characterized, ex ante, by a large amount of demand uncertainty. Thus, far more so than their established counterparts, smaller start-up firms are exposed to a serious risk: the risk of bankruptcy or firm failure, should demand turnout to be low. Consequently, although it is quite natural to assume that established firms make decisions to maximize expected profits, it is less clear that start-ups should or do behave in the same way: as Radner and Shepp (1996) and Dutta and Radner (1999) demonstrate, a firm prone to bankruptcy that purely maximizes expected profit over an infinite horizon will fail with probability one. The objective of a start-up should, then, take into account the acute risk of failure associated with entry into a new market. This implies that firms particularly prone to bankruptcy—for our purposes, start-ups entering new markets—in

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[3] In reality, firms may be able to invest in capacity in multiple periods. Allowing such an option clearly does not impact the evaluation of deferred (late) investment, though it may increase the value of early investment. If fixed costs of capacity installation or expansion are high, then the value of an option to invest in both periods is relatively low. In the extreme case, if fixed costs are high enough, then firms will only invest in capacity in one period. This is the case that we consider.

[4] We have demonstrated numerically that our core results are robust to the additional complication of holdback, i.e., producing a quantity less than the total capacity. Details are available from the authors upon request.

fact have a utility function that depends both on *operating profit* and the *risk of failure*, e.g.,

$$total\ utility = operating\ profit - cost\ of\ bankruptcy$$
$$\times probability\ of\ bankruptcy, \qquad (3)$$

where the *cost of bankruptcy* represents either real costs (e.g., default penalties on loans) or a virtual penalty term embodying the expected consequences of bankruptcy.[5] This type of utility function can be found, for example, in the seminal paper by Greenwald and Stiglitz (1990) and in Brander and Lewis (1988) and Walls and Dyer (1996). If the probability of default due to the outcome of this particular market is very low, then the firm may safely ignore the last term and simply maximize expected operating profits; this would be the case with large, established firms considering diversifying entry into a new market that represents a small potential fraction of their total business. Our model in the preceding section addressed precisely this scenario.

Alternatively, if the cost of bankruptcy is large compared to the assets of the firm and would result in financial ruin, or if the probability of bankruptcy is high (either of which is likely to be the case for a start-up), the second term dominates the expression; the maximization problem may then be thought of as approximately equal to minimizing the probability of bankruptcy or, equivalently, maximizing the probability of survival. As a result, in what follows, we assume that the presence of failure risk implies that start-ups have a different objective than established firms: instead of maximizing expected profits, they maximize their chance of survival. Essentially, although any firm has a true profit function that accounts for both operating profits and the chance of bankruptcy as depicted in (3), we examine extreme cases: established firms are entirely concerned with operating profits, whereas start-ups are entirely concerned with the probability of bankruptcy. As Chod and Lyandres (2011) discuss, the owners of private firms (e.g., start-ups in our model) are typically less diversified than the owners of public firms (established firms in our model), and hence are more sensitive to the risk inherent in a single venture and the corresponding chance of failure. Thus, it is reasonable that start-ups and established firms have different objectives; see Chod and Lyandres (2011) and references therein for a detailed discussion

of this matter. This dichotomization of the objective function, although stylized, allows us to obtain sharp results; we extend our analysis numerically to the case of other, more complicated objective functions in §6.3.

Consequently, the details of the model are identical to those introduced in §3.1, except for the objective function of the firm. We use the subscript s to denote a start-up firm. The start-up seeks to time its capacity investment and set the precise capacity level to maximize the *probability of survival*, denoted by $\psi_s(K_s)$. We assume that survival occurs for the start-up if, at the end of the selling season, total revenues are greater than debt, where debt is defined to be the sum of two components: fixed, capacity-independent debt α, and variable, capacity-dependent debt, which is linear in the installed capacity.

The fixed component of debt, α, is an exogenous parameter that may represent, for example, loans taken to fund initial start-up expenses, overhead, market research, or research and development (R&D) costs. This aspect of the start-up's debt is preexisting and fixed at the start of our model, and the terms of the loan are structured such that α must be repaid after the start-up begins generating revenues. In other words, the start-up raises capital in multiple rounds; early rounds fund R&D and start-up expenses, whereas late rounds fund capacity investment. We analyze the stage of the game after the early rounds but before the later rounds, i.e., after the start-up's initial capital structure, R&D expenses, etc., have been fixed, similar to the second stage of the two-stage capital structure and capacity games analyzed by Brander and Lewis (1986, 1988).

The variable component of debt, linear in the capacity level, is only raised at the time that capacity is installed. Regardless of when the capacity investment is made (early or late), the terms of the loan state that repayment occurs after the start-up has generated revenues, i.e., at the end of the selling season. Consequently, the start-up must generate enough operating revenue during the selling season to pay both components of its debt; otherwise, it will fail. In other words, survival occurs if *operating revenue* $\geq \alpha + capacity\ costs$, or equivalently, if operating profit (revenues minus capacity costs) is greater than the fixed debt α. The sequence of events is summarized in Figure 1.[6]

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[5] From the accounting and financial points of view, the meaning of the word "bankruptcy" is often complex and does not necessarily imply that the company fails; the actual event of bankruptcy can have varying degrees of consequence to a firm, ranging from reorganization (Chapter 11 bankruptcy) to total liquidation (Chapter 7 bankruptcy). When using this term, we simply imply that the company becomes insolvent and ceases to exist because of the negative cash flow.

[6] In reality, financing costs (and hence the cost of financed capacity and the probability of bankruptcy) would be determined in a creditor–firm equilibrium and may be a function of existing debt (α), the amount of installed capacity (K), and the default risk of the firm. Moreover, we have not addressed the case when some capacity is funded using internal equity and some capacity is paid for by financing. We make a simplifying assumption that financing costs are exogenous and all capacity is paid for by financing to obtain insights into the competitive timing game; however, analysis of the full equilibrium with internal equity and endogenous financing costs may prove to be an interesting direction for future work.

Figure 1   The Sequence of Events in the Monopolistic Start-up Model
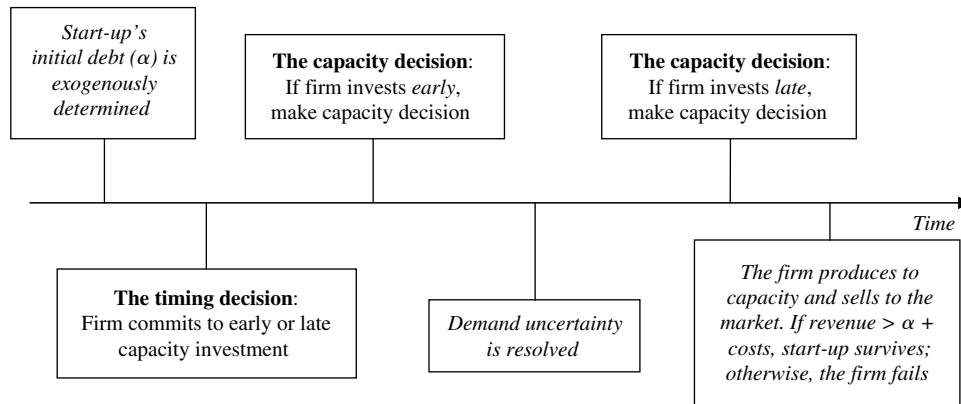
In what follows, we assume that the start-up's capacity costs are identical to the established firm analyzed in the preceding section (c_1 and c_2 for early and late investment, respectively), with the understanding that, in general, the cost of capacity may be different for a start-up, particularly if the cost of capital differs from an established firm. Section 6.2 explores a generalization of our model with heterogeneous capacity costs.

The optimal survival probability from early investment is thus

$$\psi_s^* = \max_{K_s \geq 0} \Pr((A - K_s - c_1)K_s \geq \alpha), \tag{4}$$

whereas the optimal survival probability from late investment is

$$\psi_s^* = \Pr\left(\max_{K_s \geq 0}((A - K_s - c_2)K_s) \geq \alpha\right). \tag{5}$$

Note that, in Equation (5), we have assumed that a start-up investing late, no longer subject to any uncertainty in demand, chooses a capacity level to maximize profit; at this stage, the start-up does not maximize the probability of survival because the lack of uncertainty makes this quantity ill defined. However, by maximizing profit after observing market size under late investment, the start-up survives in the largest number of demand states of any possible alternative strategy, and hence this strategy is optimal in terms of maximizing the ex ante survival probability. Also, we observe that it is possible for A to be sufficiently low that survival is impossible. In this case, the start-up still enters the market and invests in the profit-maximizing capacity despite the fact that it is doomed to failure. Because the start-up is already accountable for the initial debt, α, it cannot avoid bankruptcy by investing in zero capacity. But building the profit-maximizing capacity ensures that the start-up's lenders can be repaid to the greatest extent possible—as might be the case, e.g., if the start-up

enters bankruptcy and its assets are managed to repay as much debt as possible before liquidation.

The following theorem describes the optimal investment timing and capacity decisions, given Equations (4) and (5).

THEOREM 2. *A monopolist start-up prefers early investment if and only if $c_1 < c_2$, yielding optimal capacity $K_s^* = \sqrt{\alpha}$ and survival probability $\psi_s^* = 1 - F(2\sqrt{\alpha} + c_1)$. Otherwise, the firm prefers late investment, yielding optimal capacity $K_s^* = (A - c_2)/2$ and survival probability $\psi_s^* = 1 - F(2\sqrt{\alpha} + c_2)$.*

Theorem 2 demonstrates that a start-up prefers early investment only if costs increase over time ($c_1 < c_2$). If costs decrease over time, the start-up prefers late investment. Whereas the latter result is identical to the established firm case, the former is not; Theorem 1 shows that the established firm can prefer late investment even if costs increase over time, as long as demand uncertainty is large enough. Thus, we conclude from Theorems 1 and 2 that, given any particular set of problem parameters, a monopolistic start-up is more likely to prefer early investment than an established firm.

It is somewhat counterintuitive that a start-up, prone to such serious consequences should failure occur, is more willing to invest in capacity early than an established firm (given that the two firms have equal capacity costs); moreover, the start-up's decision is curiously unaffected by the degree of demand uncertainty. The reason for the latter result is that the start-up maximizes the probability of survival by maximizing the range of demand outcomes in which it survives. To accomplish this, it chooses the capacity that leads to survival at the lowest possible demand threshold—with this capacity level, the firm will survive for all higher demand realizations. This threshold demand level is independent of the demand variance, hence variance does not impact the start-up's survival-maximizing capacity decision.

In addition, because the start-up chooses the capacity level that ensures survival over the largest range of demand outcomes, the ability to respond to demand via late investment is not valuable to the start-up; late investment does not change the minimum demand level that ensures survival, and hence does not increase the start-up's survival probability. What *does* impact survival probability is capacity cost: lower capacity costs lead to a lower survival threshold and hence a greater survival probability. Consequently, as Theorem 2 shows, when capacity costs change over time, the survival probability will be greater in the lower cost period, which leads to the result that the start-up prefers to invest in the period with the lowest cost.

Last, we observe that the expression for the optimal capacity level under early investment, $K_s^* = \sqrt{\alpha}$, can lead to seemingly counterintuitive behavior. The fact that the optimal capacity is independent of both demand uncertainty and cost is a consequence of our stylized objective function; a more complicated (and realistic) objective function that incorporates both profit and bankruptcy risk will, in general, yield optimal capacities dependent on α, demand uncertainty, and capacity costs.

Qualitatively, the insights generated by these stylized results are compelling. For instance, if α is very small, the optimal capacity is also very small, suggesting the start-up is very risk averse for a small bankruptcy threshold; if α is very high, the optimal capacity is also large, suggesting the start-up is very risk seeking when the chance of bankruptcy is high. But a start-up maximizing the probability of survival is neither risk averse *nor* risk seeking: it is averse to bankruptcy. The optimal capacity $K_s^* = \sqrt{\alpha}$ is entirely consistent with a notion of avoiding bankruptcy: if α is small, bankruptcy can only occur if demand is very low relative to capacity, hence the optimal action (to minimize the chance of bankruptcy) is to set a very small capacity; similarly, if α is very large, survival can only occur if demand is high and the firm can capitalize on this, so the optimal action is to set a high capacity and "hope for the best." Thus, a start-up at high risk of bankruptcy (high α) can act in a seemingly aggressive manner, whereas a start-up with a low risk of bankruptcy (low α) will act more conservatively; this type of behavior will play a key role in determining the outcome of competition.

4.   Duopoly Model

We now move to the duopoly model. The details of the model are identical to the monopoly model addressed in the previous section, except there are now *two firms* competing with perfectly substitutable products in the new market. One firm is a start-up (denoted by s) and maximizes the probability of

survival, whereas the other is an established firm (denoted by e) that maximizes expected profit. The quantity of the product released to the market by firm i is Q_i, $i \in \{s, e\}$. The market price of the product is given by the linear demand curve $p(Q_i, Q_j) = A - Q_i - Q_j$. As before, A is a random variable with positive support, distribution function F, mean μ, and variance σ^2. Firms have identical capacity costs, which, as in the monopoly case, may vary over time (heterogeneous costs are discussed in §6.2). Note that we assume that neither firm is an incumbent in the market, thus a typical nomenclature in the disruptive innovation literature—entrant versus incumbent firms—does not exactly apply to our model. It might be natural, though, to assume that the established firm is an incumbent in a related market or industry. Examples of this scenario include Amazon.com and Barnes and Noble, both of whom entered the online book space at roughly the same time, despite the fact that Barnes and Noble was an "incumbent" in the related market of brick-and-mortar book retailing; and Webvan, a start-up that competed with existing traditional grocery stores in the emergent online grocery market in the early 2000s.

Before the early period (e.g., during an even earlier "decision period"), the firms simultaneously make their capacity timing decisions. Each firm has two possible actions: either commit to invest in the early period or commit to delay until the late period. We assume that these actions are credible and irreversible. This initial game is referred to as the investment timing game, or merely the *timing game*. There are four possible pure-strategy outcomes to the timing game: both firms invest early, both firms defer until the late period, and the two asymmetric outcomes in which one firm invests early and one firm invests late. The timing game and the abbreviations used to refer to its outcomes are depicted in Table 1.[7]

The *capacity subgame* then unfolds according to the sequence of moves determined by the timing game. In the late period, we assume all actions from the early period are publicly observable (e.g., if the established firm invests in the early period and the start-up

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[7] We note that while we consider a first stage investment timing game with embedded capacity subgames for its analytical convenience, this game is equivalent to a game in which firms do not first decide on an investment time, but rather simultaneously decide whether and how much to invest in the early period (i.e., whether to "invest now or wait"), under one key condition: if a firm unilaterally deviates from a particular equilibrium investment sequence, its competitor is allowed to optimally adjust capacity (but not investment timing) in response to this deviation. We believe this is a plausible scenario in reality, as capacity investment is a lengthy process and hence a firm sensing its competitor will deviate from a timing sequence (e.g., that the competitor will move from early to late investment) seems likely to modify its capacity level in the midst of the investment/construction process.

Table 1 The Four Possible Sequences of Moves and Their Abbreviations

	Established firm early	Established firm late
Start-up early	(E, E)	(E, L)
Start-up late	(L, E)	(L, L)

defers, the start-up observes the precise capacity level of the established firm at the beginning of the late period before choosing its own capacity level). Thus, in addition to the informational and cost considerations from the monopoly model, there are strategic factors in play with the timing of capacity investment: if one firm moves early and the other moves late, the early-moving firm enjoys a leadership position in a sequential game, whereas the late-moving firm is a sequential follower. As before, we assume that capacity investment is irreversible, and firms may invest in capacity in at most one period. The sequence of events is identical to the monopoly sequence in Figure 1, except the timing decision step is now a (simultaneous) game between two firms, with capacity decisions subsequently occurring in the sequence specified by the equilibrium to the timing game.

In the following four lemmas, we analyze the equilibria to each of the four capacity subgames depicted in Table 1. Once we have derived these equilibria, we may in turn analyze the equilibrium to the investment timing game. We first consider the case in which both firms invest in capacity late, i.e., after observing A. Because there is no randomness, as in the monopoly model, the start-up will choose capacity to maximize profit. The following lemma describes the equilibrium capacity investments for each firm in this game, in addition to providing the ex ante survival probability of the start-up (ψ_s^*) and the ex ante expected profit of the established firm ($\mathbb{E}(\pi_e^*)$).

Lemma 1. *If both firms invest in capacity late, then equilibrium capacities are $K_e^* = K_s^* = (A - c_2)/3$. The ex ante equilibrium expected profit of the established firm is $\mathbb{E}(\pi_e^*) = (\sigma^2 + (\mu - c_2)^2)/9$, whereas the ex ante equilibrium survival probability of the start-up is $\psi_s^* = 1 - F(3\sqrt{\alpha} + c_2)$.*

We now move to the game in which the established firm invests late and the start-up invests in capacity early, i.e., prior to observing A.

Lemma 2. *If the start-up invests early and the established firm invests late, equilibrium capacities are $K_e^* = (A - \sqrt{2\alpha} - c_2)/2$ and $K_s^* = \sqrt{2\alpha}$. The ex ante equilibrium expected profit of the established firm is $\mathbb{E}(\pi_e^*) = (\sigma^2 + (\mu - c_2 - \sqrt{2\alpha})^2)/4$, whereas the ex ante equilibrium survival probability of the start-up is $\psi_s^* = 1 - F(2\sqrt{2\alpha} + 2c_1 - c_2)$.*

We next consider the case in which both firms invest in capacity early, i.e., before observing the value of A.

Lemma 3. *If both firms invest early, equilibrium capacities are $K_e^* = (\mu - \sqrt{\alpha} - c_1)/2$ and $K_s^* = \sqrt{\alpha}$. The ex ante equilibrium expected profit of the established firm is $\mathbb{E}(\pi_e^*) = \frac{1}{4}(\mu - c_1 - \sqrt{\alpha})^2$, whereas the ex ante equilibrium survival probability of the start-up is $\psi_s^* = 1 - F(2\sqrt{\alpha} + (\mu - \sqrt{\alpha} + c_1)/2)$.*

Last, we address the case in which the start-up invests in capacity late and the established firm invests in capacity early.

Lemma 4. *If the established firm invests early and the start-up invests late, equilibrium capacities are $K_e^* = (\mu - 2c_1 + c_2)/2$ and $K_s^* = (2A - \mu + 2c_1 - 3c_2)/4$. The ex ante equilibrium expected profit of the established firm is $\mathbb{E}(\pi_e^*) = ((\mu + c_2 - 2c_1)^2)/8$, whereas the ex ante equilibrium survival probability of the start-up is $\psi_s^* = 1 - F(2\sqrt{\alpha} + (\mu - 2c_1 + 3c_2)/2)$.*

5. Equilibrium to the Timing Game

Having derived equilibria to each of the capacity investment subgames, we may now derive the equilibrium to the investment timing game. The following theorem describes all of the possible equilibria to this game.

Theorem 3. *Let $\Delta c \equiv c_1 - c_2$, let $\Sigma_1 \equiv (\mu - c_1 - \sqrt{\alpha})^2 - (\mu - c_2 - \sqrt{2\alpha})^2$, and let $\Sigma_2 \equiv \frac{9}{8}(\mu + c_2 - 2c_1)^2 - (\mu - c_2)^2$. Then the following pure-strategy equilibria to the investment timing game exist:*

1. If $\sigma^2 < \Sigma_1$ and $\Delta c < \frac{1}{3}\sqrt{\alpha}$, then both firms invest early.

2. If $\sigma^2 > \Sigma_1$ and $\Delta c < ((3 - 2\sqrt{2})/2)\sqrt{\alpha}$, then the start-up invests early and the established firm invests late.

3. If $\sigma^2 > \Sigma_2$ and $\Delta c > ((3 - 2\sqrt{2})/2)\sqrt{\alpha}$, then both firms invest late.

4. If $\sigma^2 < \Sigma_2$ and $\Delta c > \frac{1}{3}\sqrt{\alpha}$, then the start-up invests late and the established firm invests early.

There are several interesting consequences of these results. First, we note that the equilibrium regions are not exhaustive in covering the parameter space, nor are they mutually exclusive. As a result, regions of no (pure-strategy) equilibria can occur, as can regions of multiple equilibria (in particular, regions in which late investment by both firms and early investment by both firms are both possible equilibria). In all, there are six potential equilibrium regions to the investment timing game: one region each for (L, L), (E, L), (L, E), and (E, E); one region in which (E, E) and (L, L) are both possible; and one region in which no equilibrium exists. It may also be the case that the regions of (L, E) equilibrium existence and nonexistence and multiple equilibria are empty, depending on the parameter values.

To help understand the behavior described in Theorem 3, it is useful to graphically compare possible equilibrium outcomes to the monopoly case. Figure 2 does this for a typical scenario. First, note that Figure 2(a) shows the optimal investment timing for a monopolist as a function of the variance of demand (vertical axis) and the cost differential $\Delta c = c_1 - c_2$: the solid line represents the boundary between early and late investment for a profit-maximizing firm, whereas the dashed line represents the boundary for a survival-maximizing start-up. As the figure shows, the start-up prefers early investment for a much larger portion of the parameter space.

Figure 2(b) depicts the timing equilibrium regions in the competitive model using the same parameter values as Figure 2(a). The first observation one can make is that in the competitive case, early invest-

Figure 2 **(a) Optimal Investment Timing for a Monopolist; (b) Equilibria to the Investment Timing Game Between a Start-up and an Established Firm**

Notes. In panel (a), the solid line represents the boundary for an established firm, and the dashed line for a start-up firm. In both panels, $c_1 = 1$, $\alpha = 10$, and $\mu = 20$.

ment (for both firms) is far more likely. Moreover, if demand uncertainty is sufficiently high and costs do not decrease too much over time (Δc is not too large and σ^2 is not too small; case (2) of Theorem 3), the unique equilibrium to the investment timing game is for the start-up to invest early and the established firm to invest late.

This equilibrium precisely describes the situation discussed by Christensen and Bower (1996): a new market enabled by disruptive technology with highly uncertain demand, in which a start-up plays the role of leader and the established firm the role of follower. This occurs because of three competing forces in the model. The first is that early investment is valuable because of first-mover advantage in a sequential capacity game (if the competitor invests late). The second is that late investment is valuable because of the ability to exploit demand variance. The third is that the cheaper investment period is valuable because of cost savings, which can impact the value of either period. As we have already seen in the monopoly model, the second reason does not impact a start-up; hence, if costs do not decline severely over time, the start-up prefers early investment because of the leadership position in the capacity game. (Note that, unlike the monopoly model, a start-up facing competition from an established firm may invest early in capacity even if late investment is cheaper.)

By contrast, the established firm *does* value late investment because of the ability to exploit demand variance; hence, if variance is sufficiently high, the established firm prefers late investment even though it cedes a leadership position to the start-up. In particular, the start-up continues to choose the minimum capacity level that ensures survival over the widest range of demand outcomes, and hence does not exploit its leadership position to greatly increase capacity as a profit-maximizing firm might; consequently, it would appear that the established firm does not surrender as much by following a start-up as it might by following another established firm, a hypothesis that we verify in §6.1 by analyzing a model of two competing established firms.

We also observe that when costs decrease significantly over time, the picture can become complicated. In particular, a unique equilibrium may exist (either both early or both late, or the start-up following the established firm), multiple equilibria may exist, or a pure-strategy equilibrium may fail to exist. In the region of nonexistence (denoted by the null symbol in Figure 2(b)), the start-up prefers to invest at the same time as the established firm (i.e., the start-up would like to exploit cost reduction and information but only if it does not mean giving up a leadership position), whereas the established firm prefers to invest at the
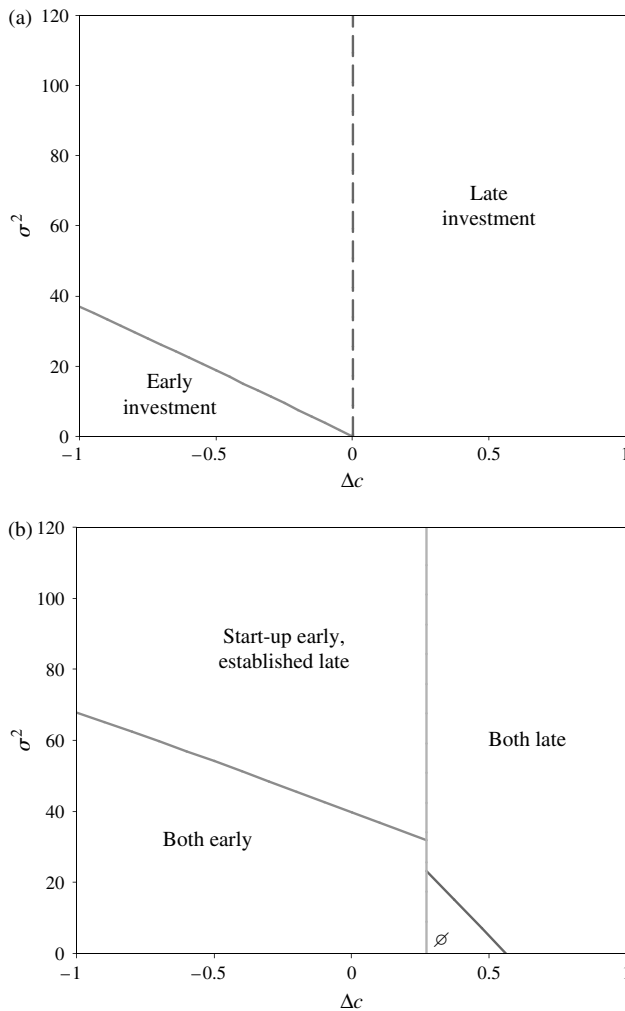
opposite time of the start-up. As a result, the outcome of the game is unclear in this region (although, it should be noted, the region of nonexistence typically covers a very small portion of the parameter space). Moreover, it is possible for an (L, E) equilibrium to exist if Δc is sufficiently large (or if α is sufficiently small) and demand uncertainty is small; however, this equilibrium never exists for the parameter values used to generate Figure 2. Indeed, the equilibrium does not exist for most reasonable parameter values, because the decline in capacity costs over time must be very large relative to the mean demand and the bankruptcy threshold α; for example, if $c_1 = 1$ and $c_2 = 0.8$, representing a 20% cost reduction from period 1 to period 2, then for (L, E) to be an equilibrium, it must be true that the bankruptcy threshold is $\alpha < 0.77$.

6. Extensions

6.1. Competition with Two Established Firms
In this section, we analyze an investment timing game identical to the one discussed in §5, with one key difference: rather than competition between a start-up and an established firm, *both* firms are established, profit-maximizing firms. We assume, as before, that the firms are ex ante identical in all other respects. This allows us to compare the outcomes of the timing game with heterogeneous firms to an otherwise identical game with two mature firms, thus isolating the impact of bankruptcy risk on capacity investment timing. The following theorem presents the equilibrium to the timing game in this case.

Theorem 4. *If two established firms compete in an investment timing game, then there exists some threshold σ^* such that, for all $\sigma > \sigma^*$, the unique equilibrium of the investment timing game is for both firms to invest late.*

As the preceding theorem demonstrates, a high degree of demand uncertainty leads to a unique equilibrium outcome when established, profit-maximizing firms compete: both firms invest in capacity late. This is in stark contrast to the investment timing equilibrium when a start-up competes with an established firm: in that case, we observed that high demand uncertainty can lead to equilibrium outcomes in which the start-up acts as a sequential leader in the investment game. We note that, in the game with two established firms, asymmetric outcomes *can* occur for lower demand variability; however, they can never occur if demand variability is sufficiently large, unlike in the model with one start-up and one established firm. Hence, we conclude that a start-up's propensity to avoid bankruptcy can have a significant effect on the dynamics of competition, particularly when demand uncertainty is high in the context of new markets.

6.2. Firms with Heterogeneous Capacity Costs
In this extension, we return to the base model (one start-up and one established firm) and consider the impact of heterogeneous capacity costs. For the sake of simplicity, we will assume that costs are constant over time for both firms, because we have already explored the impact of time-varying costs. Let the cost of the established firm be c_e, and let the cost of the start-up be c_s. Our analysis of the asymmetric capacity games in fact already accommodates heterogeneous costs (because costs in the base model vary over time, when firms invest at different times, costs are by definition heterogeneous). Thus, we need only modify our analysis to account for heterogeneous costs in the symmetric investment games. The following lemma summarizes the equilibria to the capacity investment games.

Lemma 5. *If firms have heterogeneous capacity costs that are constant over time, then:*

1. If both firms invest in capacity late (L, L), then equilibrium capacities are $K_e^* = (A + c_s - 2c_e)/3$ and $K_s^* = (A + c_e - 2c_s)/3$. The ex ante equilibrium expected profit of the established firm is $\mathbb{E}(\pi_e^*) = (\sigma^2/9) + ((\mu + c_s - 2c_e)/3)^2$, whereas the ex ante equilibrium survival probability of the start-up is $\psi_s^* = 1 - F(3\sqrt{\alpha} + 2c_s - c_e)$.

2. If the start-up invests early and the established firm invests late (E, L), equilibrium capacities, profits, and survival probabilities are identical to those derived in Lemma 2, with $c_e = c_2$ and $c_s = c_1$.

3. If both firms invest early (E, E), equilibrium capacities are $K_e^* = (\mu - \sqrt{\alpha} - c_e)/2$ and $K_s^* = \sqrt{\alpha}$. The ex ante equilibrium expected profit of the established firm is $\mathbb{E}(\pi_e^*) = \frac{1}{4}(\mu - c_e - \sqrt{\alpha})^2$, whereas the ex ante equilibrium survival probability of the start-up is $\psi_s^* = 1 - F((\mu + 3\sqrt{\alpha} + 2c_s - c_e)/2)$.

4. If the established firm invests early and the start-up invests late (L, E), equilibrium capacities, profits, and survival probabilities are identical to those derived in Lemma 4, with $c_e = c_1$ and $c_s = c_2$.

Armed with the equilibrium survival probabilities and expected profits, we may derive the equilibrium to the capacity investment timing game:

Theorem 5. *If firms have heterogeneous capacity costs that are constant over time, a unique equilibrium to the timing game exists. Let $\Sigma_1 \equiv (\mu - c_e - \sqrt{\alpha})^2 - (\mu - c_e - \sqrt{2\alpha})^2$, and let $\Sigma_2 \equiv (\mu + c_s - 2c_e)/2\sqrt{2}$. Then the following pure-strategy equilibria to the investment timing game exist:*

1. If $\sigma^2 > \Sigma_1$, the start-up invests early, whereas the established firm invests late.

2. If $\sigma^2 < \Sigma_1$ and $\sqrt{\alpha} > c_e - c_s$, both firms invest early.

3. If $\sigma^2 < \Sigma_2$ and $\sqrt{\alpha} < c_e - c_s$, the established firm invests early, whereas the start-up invests late.

Intriguingly, when costs are constant over time but differ between the two firms, only one equilibrium is possible when demand uncertainty is high: the start-up is the leader. This preserves our main result—that bankruptcy risk leads to an increased frequency of equilibria in which start-ups lead established firms—and demonstrates that it is not sensitive to the homogeneous cost assumption.

6.3. Alternative Objective Functions

Thus far, we assumed that the start-up chooses a capacity level and investment time to maximize its probability of survival. In this section, we numerically examine the impact of two alternative objective functions for a start-up. The first is referred to as the *integrated objective function* and is equal to the expected operating profit (π) minus an exogenous bankruptcy penalty (D) times the probability of bankruptcy ($1 - \psi$), i.e., $\mathbb{E}(\pi_s) - D \times (1 - \psi_s)$. This implies that the start-up cares about both profit *and* the probability of survival. As one might expect, because this objective is a linear combination of the previously analyzed survival probability and profit objectives, the behavior of a firm choosing capacity and investment time to maximize the integrated objective lies somewhere between that of a purely profit-focused and a purely survival-focused firm. In particular, the firm places more weight on the potential cost advantages of early investment (because this lowers the chance of bankruptcy) and less weight on the variance-exploiting advantages of late investment than a purely profit-maximizing firm. Consequently, depending on the precise value of D (and hence the relative weight placed on each portion of the objective function), the equilibria to the timing game resembles a mixture of the cases analyzed previously (with a survival-maximizing firm, and with two profit-maximizing firms).

The second alternative objective function is called the *limited liability objective function*. Start-ups financing their activities may be subject to limited liability should bankruptcy occur, which implies that it is only the profit above the bankruptcy threshold that truly matters (Jensen and Meckling 1976, Brander and Lewis 1986). In this scenario, the start-up is assumed to lose all profit if bankruptcy occurs (e.g., any remaining funds are distributed to debtholders) while keeping any excess profit above the survival threshold; consequently, the start-up only cares about expected profit in excess of the survival threshold, i.e., $\mathbb{E}(\pi_s - \alpha \mid \pi_s \geq \alpha) \times \psi_s$. Unlike the integrated objective function, this is not a linear combination of the profit-maximizing and survival-maximizing functions. As a result, how this objective function impacts the equilibrium to the investment timing game is, at first, not obvious.

Table 2 **Parameter Values Used in Numerical Experiments**

Parameter	Values
Demand distribution (A)	Gamma
μ	10
σ	$\{2.5, 5, 7.5, 10, 12.5, 15, 17.5, 20\}$
c_1	1
c_2	$\{0.333, 0.667, 1, 1.333, 1.667\}$
α	$\{10, 20, 30, 40, 50\}$

Although neither of these functions permits the relatively clean analytical treatment of a survival-maximizing objective function, it is possible to analyze both using numerical methods, which we have done using 200 problem instances consisting of every combination of parameters in Table 2, selected to provide a wide range of possible scenarios (e.g., low to high demand variability, various product margins, etc.). Table 3 presents the results of our numerical study. For the sake of comparison, the first row of the table lists equilibrium incidence for our base model (a survival-maximizing start-up), and the last row lists results for a model with two profit-maximizing firms. As the table shows, both the limited liability and integrated objective models yield results somewhere between the survival-maximizing and profit-maximizing cases.

The table demonstrates a key feature of our model: that bankruptcy tends to shift equilibria toward the sequential outcome with the start-up as the leader. The intuition behind this result is clear in the case of the integrated objective function, because it is a linear combination of expected profit and survival probability: later investment allows the firm to exploit demand variance, which increases the value of the profit portion of the objective function, whereas earlier investment (particularly if it is less costly) allows the firm to reduce the chance of bankruptcy and hence reduce the impact of the bankruptcy penalty. Depending on the value of the penalty parameter (D), the frequency of equilibria occurrence is somewhere

Table 3 **Incidence of Equilibria to the Investment Timing Game Under Various Models**

	Investment sequence (%)			
Model	(E, E)	(E, L)	(L, E)	(L, L)
Base model	10	69	0	24
Limited liability start-up	8	29	0	66
Integrated objective start-up, $D = 10$	12	4.5	0	88
Integrated objective start-up, $D = 100$	12	11	0	81
Integrated objective start-up, $D = 1{,}000$	12	32	0	59
Integrated objective start-up, $D = 10{,}000$	12	56	0	35
Two profit-maximizing firms	10	2.5	2.5	90

Note. Note that the total percentages of equilibrium incidence may sum to more or less than 100 because of regions of potential nonexistence and multiple equilibria.

between that of the purely profit-maximizing and purely survival-maximizing cases.

As the table demonstrates, similar to the integrated objective, the incidence of equilibria under limited liability also lie somewhere between the base survival-maximizing case and the profit-maximizing case. Compared to the profit-maximizing case, fewer (L, L) equilibria and more (E, L) equilibria occur; with limited liability, sequential outcomes (with the start-up as leader) are more likely than sequential outcomes in competition between two profit-maximizing firms.

These numerical tests show that a shift toward sequential outcomes persists regardless of the precise way in which bankruptcy risk is incorporated into the start-up's objective function. With a purely survival-maximizing start-up, there is a very strong push toward sequential outcomes; with an objective function concerned with the upside of potential profit (such as the integrated objective or the limited liability objective), this effect is tempered somewhat, but not entirely eliminated. We conclude that these results support our findings that the threat of bankruptcy—manifested in the start-up's objective function in a number of different ways—leads to a greater chance of sequential outcomes in which the start-up takes a leadership role.

7. Conclusion

Our chief goal was to analyze how the threat of bankruptcy impacts the capacity investment and timing decisions of firms entering new markets. We found that in monopoly markets, start-ups are more likely to prefer early capacity investment than profit-maximizing established firms. In competitive markets, when demand uncertainty is large, the outcome of a strategic investment timing game leads to an equilibrium in which the start-up invests early and the established firm invests late—starkly contrasting to a model with two established firms, which leads to simultaneous late investment under high demand uncertainty.

We arrived at these results despite invoking several assumptions intended to minimize the incidence of sequential equilibria. For example, in previous literature, one explanation offered for established firms failing to seize opportunities in disruptive markets is that their demand forecasts are too pessimistic or simply inaccurate. We have found, on the contrary, that even if both firms have *identical* demand forecasts, sequential equilibria arise if a start-up is present. If we incorporated pessimistic forecasts by established firms into our model, this would have the effect of decreasing the expected market size in the established firm's profit function, qualitatively preserving our results. Similarly, we assumed that both firms have

access to the technology that enables the new market at the start of the strategic investment game—in other words, no firm is playing catch-up from a technological standpoint, and both are capable of capacity investment at any time.

Because start-ups may face financial constraints that limit the maximum possible expenditure on capacity, one might reasonably suppose that it is appropriate to incorporate such a constraint into our formulation. Recall that the optimal capacity level of the start-up at either investment time is the minimum capacity level at which survival can occur—if the start-up has insufficient funds to support this capacity, then survival can never occur, and hence the survival probability is zero. Alternatively, if the start-up has more funds than necessary to support this minimum capacity level, the constraint is not binding and hence is irrelevant. Thus, at least in the survival-maximizing case, such a constraint has a very "bang-bang" impact on the model: it is either irrelevant or it reduces the survival probability to zero. A financial constraint is more meaningful if the start-up considers some combination of profit and bankruptcy costs, e.g., as in §6.3. In this case, any constraint will likely limit the value of late investment because it reduces the ability of the start-up to react to high-demand states with a high capacity level; consequently, though we do not explicitly include any financial constraints in our model, we anticipate that they would either have minimal impact on our results (in the case of survival probability maximization) or they would favor early investment even more than our current model (in the case of more complicated objective functions).

We also did not model a variety of other factors that may influence capacity investment timing. For example, greater sales may be enabled by earlier entry. Directionally, the impact of this effect is clear: it increases firm incentives to invest early. Although this would likely change the equilibrium thresholds given in Theorem 3, the qualitative impact of the start-up's survival-maximizing objective function remains (as do the consequences of acting as a first or second mover in the capacity game), implying that the strategic investment game will have a similar structure and will yield similar results.

We conclude that capacity competition involving start-ups subject to bankruptcy risk—in a variety of forms—is fundamentally different in nature from the competition between established firms, and our model offers a plausible explanation of some practically observed phenomena. Managerially, these results are important because they imply that the optimal strategic investment position differs depending on the nature of the competitor. Thus, blindly following a mantra of seizing the "first-mover advantage"

can be a perilous strategy, because any such advantage (or disadvantage) depends critically on the characteristics of the firms in the market.

Although our key findings relate to equilibrium capacity investment timing, our results also relate to the literature on disruptive innovation, which has frequently observed that start-ups tend to pioneer new markets, whereas established firms postpone investment. A variety of reasons for this phenomenon are offered: the established firms are said to be too close to and too trusting of their existing customers, who themselves are ill equipped to articulate their own changing needs, therefore causing a failure to anticipate opportunities within the existing customer base; the established firms fail to recognize and cultivate entirely new markets; internal incentives at the established firms favor the development and implementation of incremental improvement over radical change. All of these explanations imply that established firms *fail* in some crucial way that newer firms do not. By controlling for these factors in our formulation, our results imply that, although it is certainly possible that managerial failures and other reasons cited in the disruptive innovation literature can lead to established firms detrimentally ceding a leadership role to start-ups in new markets, this need not be the case; the operational reality of capacity investment under demand uncertainty, coupled with facing competition from start-ups prone to failure, offers a purely rational explanation for these outcomes.

Acknowledgments

Appendix. Proofs

PROOF OF THEOREM 1. For early investment, the profit function implied by (1) is concave and yields a unique maximum at the Cournot monopoly point, $K_e^* = (\mu - c_1)/2$. Expected profit is thus $\mathbb{E}(\pi_e^*) = (\mu - c_1)^2/4$. For late investment, the profit function implied by (2) is concave and yields a unique maximum at the Cournot monopoly point, $K_e^* = (A - c_2)/2$. Expected profit is thus $\mathbb{E}(\pi_e^*) = \mathbb{E}((A - c_2)^2/4) = (\mu - c_2)^2/4 + \sigma^2/4$. □

PROOF OF THEOREM 2. For early investment, maximizing the survival probability function in (4) is equivalent to $\psi_s^* = \max_{K_s \geq 0} \Pr(A \geq \alpha/K_s + K_s + c_1) = \max_{K_s \geq 0}(1 - F((\alpha/K_s) + K_s + c_1))$, and, consequently, this is equivalent to minimizing $(\alpha/K_s) + K_s + c_1$. This expression is convex and yields a unique minimizing capacity of $K_s^* = \sqrt{\alpha}$. The corresponding optimal survival probability is thus $\psi_s^* = 1 - F(2\sqrt{\alpha} + c_1)$. Under late investment, the start-up maximizes profit after observing A. This implies the late investment capacity level

is identical to the established firm's capacity level until late investment, i.e., $K_s^* = (A - c_2)/2$. The survival probability is thus $\psi_s^* = \Pr(((A - c_2)^2)/4 \geq \alpha) = 1 - F(2\sqrt{\alpha} + c_2)$, yielding the result. □

PROOF OF LEMMA 1. Because there is no randomness if both firms invest late, the capacity investment game is a Cournot duopoly with heterogeneous costs. Thus, the profits functiosn of the firms are $\pi_e(K_e) = (A - K_e - K_s - c_2)K_e$ and $\pi_s(K_s) = (A - K_e - K_s - c_2)K_s$. Both profit functions are concave, yielding unique best replies $K_e^*(K_s) = (A - K_s - c_2)/2$ and $K_s^*(K_e) = (A - K_e - c_2)/2$. The equilibrium capacities are found by solving for the intersection of the best replies, which yields the unique equilibrium $K_e^* = K_s^* = (A - c_2)/3$. The equilibrium profit of each firm is $\mathbb{E}(\pi_e^*) = \mathbb{E}(\pi_s^*) = \mathbb{E}((A - c_2)/3)^2 = (\sigma^2 + (\mu - c_2)^2)/9$. Recall that the start-up survives if the total profit level is above α, in other words, if $((A - c_2)/3)^2 \geq \alpha$. Thus, the ex ante survival probability of the start-up and (ex ante) equilibrium expected profit of the established firm are given by the expressions in the lemma. □

PROOF OF LEMMA 2. Recall that the best reply of the established firm is $K_e^*(K_s) = (A - K_s - c_2)/2$ when both firms invest late: this continues to hold when the start-up invests early and the established firm invests late. The start-up's profit is thus $\pi_s(K_s) = (A - K_e^*(K_s) - K_s - c_1)K_s = \frac{1}{2}(A - K_s - 2c_1 + c_2)K_s$. The survival probability is the probability that $\pi_s(K_s) \geq \alpha$, i.e., $\psi_s(K_s) = \Pr(\frac{1}{2}(A - K_s - 2c_1 + c_2)K_s \geq \alpha) = 1 - F((2\alpha/K_s) + K_s + 2c_1 - c_2)$. The maximizer of the survival probability is the minimizer of the argument of F in the above equation, i.e., $K_s^* = \sqrt{2\alpha}$, yielding the expression in the lemma when substituted into the expression for the start-up's survival probability. The established firm's profit is $\pi_e(K_e) = \frac{1}{4}(A - c_2 - \sqrt{2\alpha})^2$, and ex ante expected profit is thus given by the expected value of this expression, yielding the result in the lemma. □

PROOF OF LEMMA 3. Survival for the start-up occurs if $A \geq (\alpha/K_s) + K_e + K_s + c_1$, so the survival probability is thus $\psi_s(K_s, K_e) = 1 - F((\alpha/K_s) + K_e + K_s + c_1)$. Minimizing the the argument of F in the above expression is equivalent to maximizing the probability of survival. Thus, the start-up's optimal capacity investment is $K_s^* = \sqrt{\alpha}$, a dominant action that is independent of the established firm's capacity level. The established firm's expected profit is $\mathbb{E}(\pi_e(K_s, K_e)) = (\mu - K_e - K_s - c_1)K_e$. Substituting the equilibrium K_s^* and maximizing this concave function of K_e yields the established firm's optimal capacity, $K_e^* = (\mu - \sqrt{\alpha} - c_1)/2$. The associated expected profit of the established firm and the equilibrium survival probability of the start-up are hence the expressions in the lemma.

PROOF OF LEMMA 4. The best reply of the start-up investing late is the same as in Lemma 1, i.e., $K_s^*(K_e) = (A - K_e - c_2)/2$. Hence, the established firm's expected profit from early investment is $\mathbb{E}(\pi_e(K_e)) = (\mu - K_e - ((\mu - K_e - c_2)/2) - c_1)K_e$. Maximizing this expression yields an optimal capacity level of $K_e^* = (\mu - 2c_1 + c_2)/2$ for the established firm and, hence, $K_s^* = (2A - \mu + 2c_1 - 3c_2)/4$ for the start-up. The equilibrium expected profit of the established firm and the start-up's equilibrium survival probability are thus the expressions in the lemma. □

Proof of Theorem 3. We will examine the viability of each subgame in Table 1 individually. (i) First, let us consider the equilibrium in which the start-up invests early and the established firm follows: (E, L). This is an equilibrium if no firm has incentive to unilaterally deviate: in other words, if the established firm enjoys greater expected profit than in (E, E), and if the start-up enjoys a greater survival probability than in (L, L). From Lemmas 1 and 2, comparing the arguments of the distribution function F in each of the equilibrium survival probabilities, we see that if the established firm invests late, the start-up enjoys a (strictly) greater survival probability by investing early if $2\sqrt{2\alpha} + 2c_1 - c_2 < 3\sqrt{\alpha} + c_2$. Rearranging this expression, we see it reduces to $2\sqrt{2\alpha} < 3\sqrt{\alpha} + 2(c_2 - c_1)$. If $c_1 < c_2$, the condition holds if $\alpha > 0$. If, on the other hand, $c_1 > c_2$, the start-up *may* unilaterally deviate from (E, L) for some $\alpha > 0$. Examining this expression, we see that the inequality is most likely to hold if α is large—hence, the start-up will deviate from (E, L) if costs decrease over time and α is sufficiently small. Next, consider the established firm, which, from Lemmas 2 and 3, will not deviate from (E, L) if $(\sigma^2 + (\mu - c_2 - \sqrt{2\alpha})^2)/4 > ((\mu - c_1 - \sqrt{\alpha})^2)/4$. This expression reduces to

$$\sigma^2 > (\mu - c_1 - \sqrt{\alpha})^2 - (\mu - c_2 - \sqrt{2\alpha})^2. \qquad (6)$$

In other words, the established firm will not unilaterally deviate from (E, L) if demand is variable enough, where the threshold variability is a function of the problem parameters. (ii) We next consider the equilibrium in which both firms build capacity early: (E, E). From Lemmas 2 and 3, the established firm will not deviate from this equilibrium precisely if (6) is violated. From Lemmas 3 and 4, the start-up will not deviate if $2\sqrt{\alpha} + \frac{1}{2}\mu - c_1 + \frac{3}{2}c_2 > 2\sqrt{\alpha} + (\mu - \sqrt{\alpha} + c_1)/2$. This inequality reduces to $\frac{1}{3}\sqrt{\alpha} > c_1 - c_2$. (iii) We next consider the equilibrium with both firms building capacity late: (L, L). In this case, part (i) of the proof demonstrated that the start-up prefers (L, L) to (E, L) if $2\sqrt{2\alpha} > 3\sqrt{\alpha} + 2(c_2 - c_1)$. Similarly, from Lemmas 1 and 4, the established firm will not deviate from this equilibrium if $((\mu + c_2 - 2c_1)^2)/8 < (\sigma^2 + (\mu - c_2)^2)/9$. This inequality reduces to $\frac{9}{8}(\mu + c_2 - 2c_1)^2 - (\mu - c_2)^2 < \sigma^2$. (iv) Last, we consider (L, E). The start-up has incentive to deviate from (E, E) to (L, E) if $\frac{1}{3}\sqrt{\alpha} < c_1 - c_2$, and the established firm has incentive to deviate from (L, L) to (L, E) if $\frac{9}{8}(\mu + c_2 - 2c_1)^2 - (\mu - c_2)^2 > \sigma^2$. $\square$

Proof of Theorem 4. We must first analyze several additional aspects of the capacity subgames to analyze the investment timing game. First, consider the game in which both firms invest early. This is a Cournot duopoly, hence the equilibrium profits of the (symmetric) established firms are both $\mathbb{E}(\pi_e^*) = (\mu - c_1)^2/9$. Next, consider the game in which the firms invest sequentially. This is identical to the previously analyzed game in which the established firm invests early and the start-up invests late (because, in that case, the start-up maximized profit due to the elimination of uncertainty). Hence, the profit of the leader is $\mathbb{E}(\pi_e^*) = ((\mu + c_2 - 2c_1)^2)/8$, whereas the profit of the follower is $\mathbb{E}(\pi_e^*) = (4\sigma^2 + (\mu + 2c_1 - 3c_2)^2)/16$. Finally, the game in which both firms invest late yields identical profits to both

firms equal to $\mathbb{E}(\pi_e^*) = (\sigma^2 + (\mu - c_2)^2)/9$. Thus, the investment timing game in normal form has payoffs as follows.

	Firm 2 early	Firm 2 late
Firm 1 early	$\left(\dfrac{(\mu - c_1)^2}{9}, \dfrac{(\mu - c_1)^2}{9} \right)$	$\left(\dfrac{(\mu + c_2 - 2c_1)^2}{8}, \right.$ $\left. \dfrac{4\sigma^2 + (\mu + 2c_1 - 3c_2)^2}{16} \right)$
Firm 1 late	$\left(\dfrac{4\sigma^2 + (\mu + 2c_1 - 3c_2)^2}{16}, \right.$ $\left. \dfrac{(\mu + c_2 - 2c_1)^2}{8} \right)$	$\left(\dfrac{\sigma^2 + (\mu - c_2)^2}{9}, \right.$ $\left. \dfrac{\sigma^2 + (\mu - c_2)^2}{9} \right)$

First, assume that Firm 2 invests early. Firm 1 prefers *late* investment if $(4\sigma^2 + (\mu + 2c_1 - 3c_2)^2)/16 > (\mu - c_1)^2/9$. Clearly, as σ^2 increases, this inequality is more likely to hold. Similarly, if Firm 2 invests late, Firm 1 prefers late investment if $(\sigma^2 + (\mu - c_2)^2)/9 > (\mu + c_2 - 2c_1)^2/8$. Again, as σ^2 increases, this inequality is more likely to hold, thus for large enough σ^2 (i.e., σ^2 above some threshold), late investment is the dominant strategy of both firms, and (L, L) is the only possible equilibrium. $\square$

Proof of Lemma 5. The proof has been omitted and is similar to those for Lemmas 1–4. $\square$

Proof of Theorem 5. Similar to the proof in the base model, we will examine each possible equilibrium individually. (i) (E, L) is an equilibrium if no firm has incentive to unilaterally deviate: from Lemma 5, the start-up will not deviate if $1 - F(2\sqrt{2\alpha} + 2c_s - c_e) > 1 - F(3\sqrt{\alpha} + 2c_s - c_e)$, which always holds. The equilibrium is supportable if the established firm has no incentive to deviate, i.e., if $(\sigma^2 + (\mu - c_e - \sqrt{2\alpha})^2)/4 > \frac{1}{4}(\mu - c_e - \sqrt{\alpha})^2$, which holds if $\sigma^2 > (\mu - c_e - \sqrt{\alpha})^2 - (\mu - c_e - \sqrt{2\alpha})^2$; thus, with constant, heterogeneous costs, (E, L) is an equilibrium if σ^2 is sufficiently large. (ii) The sequence (E, E) is only an equilibrium if the established firm has no incentive to deviate, which the analysis of (E, L) showed occurs for low σ^2. It must also be the case that the start-up has no incentive to deviate, which holds if $1 - F(2\sqrt{\alpha} + \frac{1}{2}\mu - c_e + \frac{3}{2}c_s) > 1 - F((\mu + 3\sqrt{\alpha} + 2c_s - c_e)/2)$, which is equivalent to $\sqrt{\alpha} > c_e - c_s$. (iii) For (L, E), the start-up has incentive to remain in this equilibrium if $\sqrt{\alpha} < c_e - c_s$. The established firm has incentive to remain in this equilibrium if $\sigma^2/9 + ((\mu + c_s - 2c_e)/3)^2 < ((\mu + c_s - 2c_e)^2)/8$. This reduces to $\sigma < (\mu + c_s - 2c_e)/(2\sqrt{2})$. (iv) The sequence (L, L) is only an equilibrium if the start-up has no incentive to deviate, which the analysis of (E, L) showed is never true. Hence, (L, L) cannot be an equilibrium. $\square$

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