Marketing Science

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To cite this article:
Gérard P. Cachon, Pnina Feldman (2017) Is Advance Selling Desirable with Competition?. Marketing Science 36(2):214-231.
http://dx.doi.org/10.1287/mksc.2016.1006

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http://pubsonline.informs.org/journal/mksc/

MARKETING SCIENCE
Vol. 36, No. 2, March–April 2017, pp. 214–231
ISSN 0732-2399 (print), ISSN 1526-548X (online)

Is Advance Selling Desirable with Competition?

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Received: May 18, 2013
Revised: May 26, 2015; May 3, 2016
Accepted: July 8, 2016
Published Online in Articles in Advance: January 19, 2017

https://doi.org/10.1287/mksc.2016.1006

Abstract. It has been shown that a monopolist can use advance selling to increase profits. This paper documents that this may not hold when a firm faces competition. With advance selling a firm offers its service in an advance period, before consumers know their valuations for the firms' services, or later on in a spot period, when consumers know their valuations. We identify two ways in which competition limits the effectiveness of advance selling. First, while a monopolist can sell to consumers with homogeneous preferences at a high price, this homogeneity intensifies price competition, which lowers profits. However, the firms may nevertheless find themselves in an equilibrium with advance selling. In this sense, advance selling is better described as a competitive necessity rather than as an advantageous tool to raise profits. Second, competition in the spot period is likely to lower spot period prices, thereby forcing firms to lower advance period prices, which is also not favorable to profits. Rational firms anticipate this and curtail or eliminate the use of advance selling. Thus, even though a monopolist fully exploits the practice of advance selling, rational firms facing competition either mitigate it or avoid it completely.

History: Preyas Desai served as the editor-in-chief and Yuxin Chen served as associate editor for this article.
Supplemental Material: The online appendix is available at https://doi.org/10.1287/mksc.2016.1006.

Keywords: consumer behavior • game theory • pricing • competitive analysis

1. Introduction

Sometimes consumers purchase a good or service well before they actually consume it. This can involve some risk—the consumers may have an expectation that they will want or need the service, but they can be uncertain in what its actual value will be at the time of consumption. There are different ways in which valuation uncertainty may arise when purchasing consumer goods or services. For example, customers may be uncertain about their value of a concert or a musical several weeks before the show or about their desire to eat at a popular restaurant in the future. Customer value may also be uncertain for a new or innovative product (e.g., an Apple Watch, Jawbone's UP3 activity tracker, Sony's VR headset, or Windows 10), an experience item (such as movies, books, or video games), or if their need for an item is uncertain (e.g., a dress for a friend's wedding they may or may not be able to attend). Common to all these examples is that customers gain a better sense of their value for the product over time. Therefore, consumers can wait to resolve that uncertainty by delaying their purchasing decision to shortly before consumption.

Firms who are aware of consumers' value uncertainty may try to exploit it by selling in advance. We say a firm *advance sells* when the firm allows consumers to buy well in advance of their consumption. We say that the firm *spot sells* when the firm sells to consumers at the time of consumption, such as selling movie tickets just before the movie begins. It has been shown that monopolies can earn more revenue by advance selling than by selling exclusively on the spot (e.g., Xie and Shugan 2001). When consumers purchase in advance, they are uninformed and are willing to pay at most their expected value for the service. By contrast, when consumers purchase on the spot, they know their exact values for the product. These values are generally different across consumers–some customers value the product more than others and no consumer is willing to pay more than his realized value. That is, in the spot period, consumers are less homogeneous relative to the advance period. Monopolistic firms can take advantage of consumers' ex ante homogeneity: A firm can earn higher revenue by selling in advance to *all* consumers (but, at a lower price) than by selling on the spot to only a portion of consumers (those who discover they have a high value for the product).

Building on the advance selling results for a monopoly, in this paper we seek to understand whether advance selling is also desirable in a competitive environment. Competition complicates the consumers' purchasing options in that under competition individual consumers decide not only on *when* and *whether* to purchase the product but also *which* firm to buy

from. This paper demonstrates that while a monopolist benefits from selling in advance, advance selling does not necessarily benefit a firm facing competition. We show that advance selling may be limited or completely eliminated in equilibrium. Even when advance selling occurs in equilibrium, firms facing competition in advance would in most cases benefit if they could commit to sell only on the spot. There are several reasons for our finding, which we illustrate using two related models. First, as consumers are more homogeneous in the advance period, they are more sensitive to the firms' advance period prices. This intensifies the competition between the firms in the advance period, driving down prices in the advance period, making advance selling less desirable. Second, competition in the spot period lowers spot period prices. As consumers only buy in advance if they get a sufficiently good discount relative to the spot period price (to compensate them for the fact that they face valuation uncertainty if they buy in advance), spot period competition forces firms to lower their advance period price even if there is no competition in the advance period. Third, selling in advance removes relatively loyal customers from the spot period, which intensifies spot period competition, which also lowers prices. Again, this is undesirable for advance selling.

2. Related Literature

Our work belongs in the intertemporal pricing literature. These papers consider pricing over multiple periods with forward-looking consumers who make dynamic choices. In markets with durable goods, consumers may time their purchase in anticipation of markdowns, e.g., Coase (1972), Stokey (1981), Besanko and Winston (1990). In all of those papers, consumers never consider purchasing when their valuation for the service is uncertain, so purchasing early (i.e., in "advance") does not involve the risk of purchasing something that is not desired later on. Other reasons for consumers to time their purchase include product availability (e.g., Su 2007, Elmaghraby et al. 2008, Aviv and Pazgal 2008, and Cachon and Swinney 2009) and product innovation (e.g., Dhebar 1994 and Kornish 2001). Our model does not include inventory decisions, or products that are changed over time.

A number of papers focus specifically on advance selling strategies. Gale and Holmes (1993) and Degraba (1995) each consider a monopoly firm with capacity constraints. Gale and Holmes (1993) show that advance-purchase discounts allow a firm to price discriminate between consumers who are reasonably certain of their future utility for a service and those who are more unsure. Degraba (1995) finds that a firm may benefit from intentional scarcity strategies that induce consumers to buy in advance to avoid a rationing risk. In our models, there are no capacity constraints, so customers' incentive to purchase in advance is due to an advance price discount rather than limited availability. Xie and Shugan (2001) extend the work of Shugan and Xie (2000) and show that a firm can be better off by selling in advance even when there are no capacity constraints as long as the unit marginal cost is not too high. For simplicity, we assume the unit marginal cost is zero. Nasiry and Popescu (2012) consider advance selling to consumers who experience regret, whereas we work in an expected revenue maximization framework. Cachon and Feldman (2011) study subscription pricing, which can be a form of advance selling applied to repeat purchases, whereas in our model consumers make a single purchase.

Several papers consider selling strategies that help mitigate consumer risks from advance purchases. For example, firms may offer advance sales that are at least partially refundable, e.g., Xie and Gerstner (2007), Guo (2009), and Gallego and Sahin (2010). These papers focus on how much refund, if any, the firm should offer customers. In these papers, advance selling may be beneficial because of the ability to sell the same unit of capacity twice. We do not consider partial refunds, as the benefits of partial refunds have been shown only in environments with limited capacity. Other papers consider strategies to help customers mitigate other types of risk. When there is limited capacity and customers are risk averse, Png (1989) shows that offering reservations before consumers learn their valuations insures them against the possibility of being rationed. In our model, customers are risk neutral, but the firm needs to provide a sufficient discount to convince forward-looking consumers to buy in advance when their value is uncertain. Firms may also offer price guarantees that give customers a refund if they find the product at a lower price elsewhere, e.g., Png and Hirshleifer (1987) and Jain and Srivastava (2000). In our paper there is no price uncertainty. We assume that customers are capable of correctly anticipating firms' spot prices.

Although most of the advance selling literature assumes a market with a single firm, there are some that consider competition. Dana (1998) demonstrates that price-taking firms may offer an advance purchase discount in a market with capacity constraints and rationing. However, in his setting, firms earn zero profit whether firms sell in advance or not, as the market is perfectly competitive. Hence, firms neither benefit nor are harmed by advance selling. Shugan and Xie (2005, p. 351) find that "competition does not diminish the advantage of advance selling." In their models, the firms sell either in advance or on the spot but not in both periods. Hence, consumers do not trade off buying in advance versus on the spot. In our model, firms sell in both periods and consumers are forward-looking. Therefore, competitiveness in the spot period

reduces spot period prices and influences the firms' advance sales. Finally, Guo (2009) considers advance selling in an oligopolistic market with capacity constraints, but as already mentioned, he focuses on the impact of including partial refunds or not.

3. Model Setup

To motivate the models, consider the following anecdotes:

- For their son's birthday in May, a family considers visiting an amusement park in Orlando, Florida. While they have been to amusement parks in the past, they have not visited the ones in Florida, and debate on which park to go to. They may buy their tickets now, or wait until April, when their friends return from a similar trip and can share their experience.
- A married couple is interested in going on a vacation to celebrate their anniversary either scuba diving on an island (e.g., in Maui) or skiing in the mountains. They are not sure which to choose in advance because they have not done scuba before and do not know if they like it. They may reserve their vacation now or wait until they complete an introductory scuba diving class.
- Apple is designing an innovative smart watch. Customers loyal to Apple have no interest in another firm's watch. They may preorder the watch now, uncertain of their value for it, or wait until it becomes available to the general public, at which time nonloyal consumers may also be interested in buying it.
- Golden State Warriors fans are debating whether to purchase basketball tickets early, fearing that they may have other commitments when game day arrives. Travelers to the Bay Area who are looking for something entertaining to do during their visit are considering a few alternative options (e.g., they may go to a concert instead), and will purchase tickets closer to the date, once they arrive in the area.

The first two examples describe situations where customers are initially uncertain regarding the firm they prefer and firms therefore engage in advance period competition. The uncertainty is resolved later so each firm is a monopoly in the spot period. The last two anecdotes are examples in which a firm is a monopoly in advance serving only its loyal customers, but competes on the spot for additional customers. Using two different models that fit these examples, we demonstrate that no matter the type of competition, advance selling is rarely desirable for competing firms. To illustrate this clearly, in both models we make the smallest possible departures from the monopolistic advance selling model in the literature (e.g., Xie and Shugan 2001) that enable competition. Here, we describe the modeling framework common to both models and later (Sections 4 and 5) describe the characteristics specific to each one.

Two firms compete in a duopoly market that spans two periods—the advance period and the spot period. The firms simultaneously set advance and then spot period prices to maximize expected revenues. A market of forward-looking consumers that rationally anticipate future prices decides whether to buy in the advance period or wait for the spot period. These consumers are uncertain about their product valuation in the advance period, but this uncertainty is resolved at the start of the spot period. Customers have independent valuations.

Both the firms and consumers are risk neutral and therefore all calculations and decisions are based on expectations (expected revenues for the firms and expected surplus for consumers). The firms have sufficient capacity to sell to all consumers.[1] For each model, we seek to characterize the subgame perfect Nash equilibria of the game. An equilibrium consists of the optimal actions chosen by the firms and the consumers given their beliefs about the actions taken by the other players. Moreover, the beliefs of all players are consistent with the equilibrium outcome.

4. Model I: Advance Period Competition

A market of consumers, which can be normalized to size 1 without loss of generality, decides which firm to buy from and in which period. In the advance period, consumers are uncertain about their consumption utility in the spot period. They face two types of uncertainty. First, they are unsure as to which firm they will prefer. Second, they are unsure as to how strongly they will value their preferred firm. Specifically, consumers know in the advance period that they will receive value V from their most preferred firm, where V can take on one of two values with equal probability, $V \in \{v_l, v_h\}, v_h \geq v_l > 0$. Consumers also know that they receive zero value from the other, nonpreferred, firm.[2] Finally, consumers know in the advance period that they will learn in the spot period which firm they prefer and how strong that preference is. Mayzlin (2006) and Guo (2009) use similar models of consumer uncertainty under competition.

The expected value of the preferred product is $\mathbb{E}[V] = (v_l + v_h)/2$. We define $\beta = v_l/v_h$. That is, β measures the value of v_l relative to v_h. As β increases, v_l approaches v_h. We assume $\beta \in [0, 1/2]$, which implies that $v_h \geq 2v_l$. Consequently, a firm's optimal spot price is v_h: a firm prefers to charge a high spot price (v_h) and sell to half of the consumers than to sell to all consumers at a low price. Thus, conditional on not owning a unit, only customers that have a high realized value for the product purchase the unit in the spot period. We do not include cases in which $v_h < 2v_l$ because these are not interesting—in those cases there is no advantage to advance selling even for a monopolist: The optimal spot price is v_l, so forward-looking consumers would

not pay more than v_l in the advance period, meaning that in all cases the monopolist sells to all consumers at v_l.

Even though consumers do not know exactly which firm they prefer in the advance period, they are also not entirely ignorant. Consumers have some information about their future spot period preference. This information can be obtained, for example, from consumers' previous experience with the firms or by reading expert reviews or blogs about the products. We model the degree of information each consumer has explicitly, by defining α as the probability that a customer attaches for preferring firm 1 in the advance period after processing the information. Therefore, $1-\alpha$ is the probability that the same consumer attaches for preferring firm 2. We allow this information to differ among consumers by assuming that $\alpha \sim U[(1-\delta)/2, (1+\delta)/2]$. Hence, there is a continuum of consumer types and a consumer's type is the probability that she will prefer firm 1. Some consumers attach a higher probability to preferring firm 1 than others.[3] When $\delta = 0$, consumers have no information with respect to which firm they will prefer and all consumers attach the same probabilities for preferring each firm (50/50). As δ increases, consumers become more heterogeneous in their knowledge. Some consumers are very informed about their firm preference, whereas others are not. This information helps consumers to better evaluate their eventual preference, before making their advance period purchasing decisions. The introduction of δ allows us to examine the effect that the heterogeneity in information has on firms' revenues and consumers' purchasing decisions in equilibrium, which we discuss in Section 4.3.

Gale and Holmes (1993) also have consumers who are heterogeneous in the advance period. In their model, a single firm offers two services. Consumers differ in the values they assign to these services but they are equally knowledgeable about which service they prefer. By contrast, in our model consumers have the same value distribution across the available services/firms but differ in their ability to identify which firm they will prefer, i.e., some may be reasonably sure which they will prefer whereas others are not. Dana (1998) allows consumers to vary in the value distribution for a service, but since all firms offer an equivalent service, consumers do not have firm preferences.

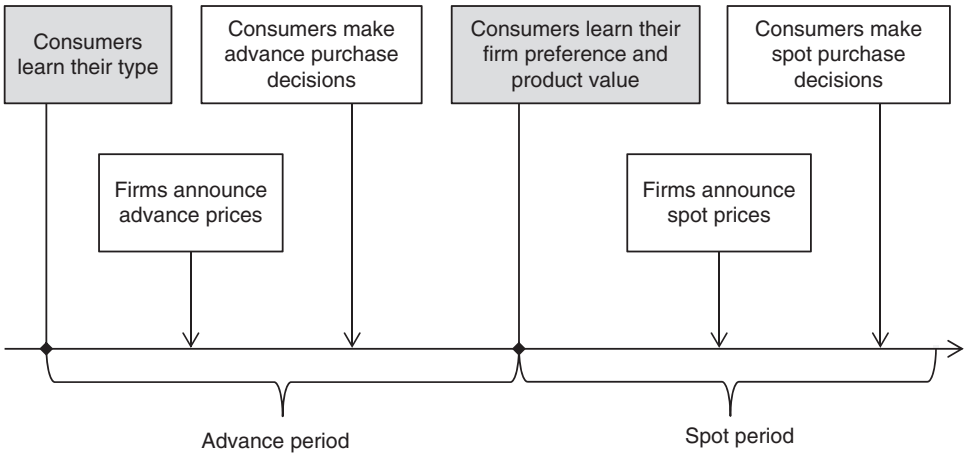
Without observing consumers' types, but knowing the distributions of V and $\boldsymbol{\alpha}$, the firms simultaneously announce advance prices p_1 and p_2. Consumers learn their types α, and decide whether to purchase in advance (and if so, from which firm) or to wait for the spot period. Being forward-looking, customers can correctly anticipate that the spot period prices are equal to v_h. We assume that consumers purchase only one unit, even if they realize that they end up preferring the other unit.[4] We believe that assuming that customers purchase only once is more realistic for the type of products that are usually sold in advance: Customers that hold tickets for a concert do not usually purchase other tickets, even if they realized that they do not value the tickets as much as they expected. This assumption is common to the literature on advance selling and competition (Shugan and Xie 2005, Guo 2009). Figure 1 demonstrates the sequence of events. Note, the firms face competition in the advance period, but they are monopolists in the spot period.

In this model, we restrict attention to symmetric subgame perfect Nash equilibria, where both firms choose the same prices in equilibrium. To identify the equilibrium, in Sections 4.1 and 4.2 we analyze consumers' behavior, given their beliefs with respect to the spot prices, and the firms' advance and spot period prices, given their beliefs about customers' purchasing decisions and the competitor's prices.

4.1. Consumer Purchasing Decisions

After receiving the signals and observing the advance prices, p_1 and p_2, announced by both firms, consumers

Figure 1. Sequence of Events in Model I



decide whether to purchase in advance and, if so, from which firm. According to the equilibrium concept, all customers share the same beliefs about the firms' spot prices and the behavior of other consumers. Moreover, because customers are forward-looking, they can correctly anticipate that each firm will charge v_h in the spot period and that they will obtain no surplus from purchasing on the spot. The next result follows. (The proofs of this and all subsequent results are given in the appendix.)

Lemma 1. *If firm* 1 *advance sells in equilibrium then there exists a unique threshold,* $\bar{\alpha}$*, such that only consumers with* $\alpha \geq \bar{\alpha}$ *buy from it in advance. Similarly, if firm* 2 *advance sells in equilibrium then there exists a unique threshold,* $\underline{\alpha}$*, such that only customers with* $\alpha < \underline{\alpha}$ *buy from it in advance.*

Lemma 1 enables us to simplify the search for consumer actions in equilibrium. Instead of analyzing each consumer's optimal purchasing decision, we can restrict attention to finding the equilibrium information thresholds that are induced by the firms' advance and spot prices. Let $\mathbf{p}$ be a vector of advance and spot prices charged by both firms. The thresholds $\bar{\alpha}(\mathbf{p})$ and $\underline{\alpha}(\mathbf{p})$ are the consumers' best responses to the firms' prices. In what follows, we analyze these best responses for a given set of prices, $\mathbf{p}$. Because in any equilibrium the firms' spot prices are v_h, we can restrict attention to best responses to the advance period prices p_1 and p_2 (and spot prices v_h).

All consumers are expected utility maximizers and therefore choose the strategy that maximizes their total expected surplus, i.e., the expected surplus of advance and spot purchases. In this model, we focus on pure strategies and assume that in the case of indifference between purchasing in the two periods, customers purchase in advance. Thus, a consumer that attaches a probability α for preferring firm 1, evaluates the expected utility of three different strategies:

1. Buy in advance from firm 1, which yields an expected utility of $\alpha\,\mathbb{E}[V] - p_1$.

2. Buy in advance from firm 2, which yields an expected utility of $(1-\alpha)\,\mathbb{E}[V] - p_2$.

3. Wait for the spot and then, if $V = v_h$, buy from the preferred firm, which yields an expected utility of zero.

We refer to a customer who obtains the same surplus by choosing two different strategies as an indifferent consumer. To buy in advance from firm 1, a customer who attaches a probability α for firm 1, must prefer to purchase in advance from firm 1 over firm 2, which happens if and only if $\alpha\,\mathbb{E}[V] - p_1 \geq (1-\alpha)\,\mathbb{E}[V] - p_2$; at the same time, this customer must prefer to purchase in advance from firm 1 rather than wait for the spot period to make her purchasing decision, which is the case if and only if $\alpha\,\mathbb{E}[V] - p_1 \geq 0$. Thus, for a consumer to buy unit 1 in advance, the attached probability, α, for preferring firm 1 should satisfy

$$\alpha \geq \max\left\{\frac{p_1}{\mathbb{E}[V]}, \frac{1}{2}\left(1 + \frac{p_1 - p_2}{\mathbb{E}[V]}\right)\right\}. \tag{1}$$

Similarly, to buy in advance from firm 2, α should satisfy

$$\alpha \leq \min\left\{1 - \frac{p_2}{\mathbb{E}[V]}, \frac{1}{2}\left(1 + \frac{p_1 - p_2}{\mathbb{E}[V]}\right)\right\}. \tag{2}$$

Combining conditions (1) and (2), we get that if $p_1 + p_2 \leq \mathbb{E}[V]$, all customers purchase in advance from either firm 1 or firm 2. Denote by $\hat{\alpha}$ the probability of the consumer indifferent between the firms. This probability is given by

$$\hat{\alpha}(p_1, p_2) = \frac{1}{2}\left(1 + \frac{p_1 - p_2}{\mathbb{E}[V]}\right).$$

Thus, all customers with $\alpha \geq \hat{\alpha}$ purchase from firm 1 in advance and all customers with $\alpha \leq \hat{\alpha}$ purchase from firm 2 in advance. If, however, $p_1 + p_2 > \mathbb{E}[V]$, some consumers buy in advance, while others wait for the spot to make their purchasing decision. Let $\bar{\alpha}$ be the preference probability of the consumer who is indifferent between buying in advance from firm 1 and waiting for the spot and $\underline{\alpha}$ be the preference probability of the consumer who is indifferent between buying in advance from firm 2 and waiting for the spot, where

$$\bar{\alpha}(p_1) = \frac{p_1}{\mathbb{E}[V]}$$

and

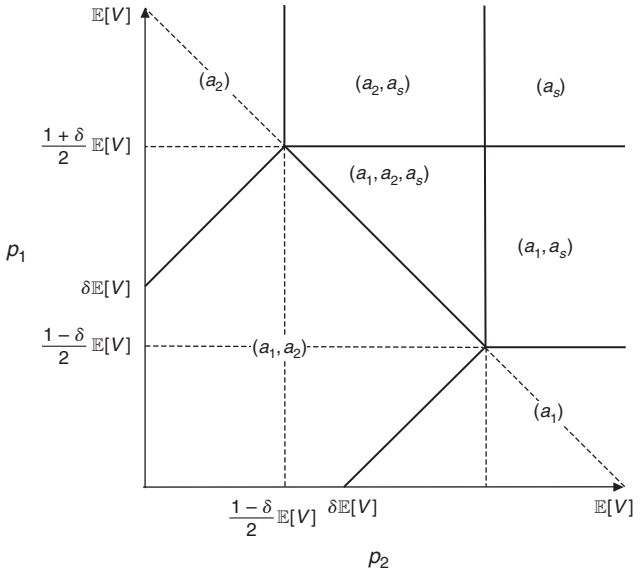
$$\underline{\alpha}(p_2) = 1 - \frac{p_2}{\mathbb{E}[V]}.$$

(Observe that $\underline{\alpha} < \bar{\alpha}$, if $p_1 + p_2 > \mathbb{E}[V]$.) Then, all consumers with $\alpha \geq \bar{\alpha}$ purchase in advance from firm 1, all consumers with $\alpha \leq \underline{\alpha}$ purchase in advance from firm 2, and all consumers with $\alpha \in (\underline{\alpha}, \bar{\alpha})$ wait for the spot.

Note that the advance period is analogous to a Hotelling (1929) line model of competition. Each firm is located at the endpoints of a segment of unit length and consumers are located uniformly along the interior segment $[(1-\delta)/2, (1+\delta)/2]$. Each customer receives a base value of $\mathbb{E}[V]$ from getting the unit and incurs a marginal travel cost of 1. Of course, the classic Hotelling line model is not concerned with dynamically pricing products across periods and the attractiveness of advance selling strategies, which is the focus of this paper.

Figure 2 demonstrates the consumer equilibrium behavior on the advance price space. The possible purchasing decisions in equilibrium are represented by $a_i, i \in \{1, 2, S\}$, where 1 denotes buying in advance from firm 1, 2 denotes buying in advance from firm 2, and

Figure 2. Customers' Behavior in a (p_1, p_2) Price Space for $\delta > 1/3$



Note. An a_i for a particular (p_1, p_2), denotes that there are consumers that follow purchasing strategy i, for that price tuple, $i \in \{1, 2, S\}$.

S denotes waiting for the spot. A particular customer's purchasing decision depends on her α in the manner explained above. For example, the area represented by (a_1, a_S) implies that consumers with $\alpha \geq \bar{\alpha}$ buy in advance from firm 1, consumers with $\alpha < \bar{\alpha}$ wait for the spot to make their purchasing decision, and no customer buys from firm 2 in advance. Figure 2 represents the equilibrium behavior for $\delta > 1/3$. With $\delta < 1/3$ the same seven regions exist and the only difference is that $(1-\delta)\mathbb{E}[V]/2 > \delta\mathbb{E}[V]$.

4.2. Firms' Revenue Functions

From the analysis in the previous section, the firms can rationally conclude how a tuple of advance prices (p_1, p_2) affects customers' purchasing decisions. That is, given a set of advance period prices firms can correctly predict their expected demand in each period and consequently their expected revenues. If $p_1 + p_2 \leq \mathbb{E}[V]$, advance period demand is $\bar{F}(\hat{\alpha}(p_1, p_2))$ from firm 1 and $F(\hat{\alpha}(p_1, p_2))$ from firm 2, where $F(\cdot)$ is the cumulative distribution function of a uniform random variable on $[0, 1]$ and $\bar{F}(\cdot) = 1 - F(\cdot)$. In this case all customers purchase in advance, so demand on the spot is 0. If, on the other hand, $p_1 + p_2 > \mathbb{E}[V]$, advance period demand is $\bar{F}(\bar{\alpha}(p_1))$ from firm 1 and $F(\underline{\alpha}(p_2))$ from firm 2. The expected spot period demand in this case is composed of customers who did not purchase in the advance period and whose realized spot value is $V = v_h$, which occurs with probability (1/2). Therefore, the expected spot period demand from firm 1 is given by

$$D_1^S = \int_{\underline{\alpha}}^{\bar{\alpha}} \frac{\alpha}{2} f(\alpha)\, d(\alpha). \tag{3}$$

Similarly,

$$D_2^S = \int_{\underline{\alpha}}^{\bar{\alpha}} \frac{1-\alpha}{2} f(\alpha)\, d(\alpha) \tag{4}$$

is the expected spot period demand from firm 2.

Table 1 lists firm 1 and firm 2 revenue functions, $\Pi_1(p_1; p_2)$ and $\Pi_2(p_2; p_1)$, for each case of customers' purchasing decisions, where D_1^S and D_2^S are the spot period expected demands in (3) and (4). Observe that for each customer behavior case, firm i's revenue function is concave in p_i. However, while the profit function is locally concave, it is not, in general, globally concave. To see this, fix p_2. As p_1 increases, customers' behavior changes and the profit function switches from one region to another. For example, for $p_2 \in (\delta\mathbb{E}[V], (1+\delta)\mathbb{E}[V]/2)$, customers' behavior changes from $(a_1) \to (a_1, a_2) \to (a_1, a_2, a_S) \to (a_2, a_S)$ with an increase

Table 1. Firms' Total Expected Revenues as a Function of Customers' Purchasing Behavior and Prices

	$\Pi_1(p_1; p_2)$	$\Pi_2(p_2; p_1)$
(a_1, a_2)	$\frac{p_1}{2\delta}\left(\delta - \frac{p_1 - p_2}{\mathbb{E}[V]}\right)$	$\frac{p_2}{2\delta}\left(\delta - \frac{p_2 - p_1}{\mathbb{E}[V]}\right)$
(a_1, a_2, a_S)	$\frac{p_1}{\delta}\left(\frac{1+\delta}{2} - \frac{p_1}{\mathbb{E}[V]}\right) + \frac{2\mathbb{E}[V]}{\beta+1} D_1^S$	$\frac{p_2}{\delta}\left(\frac{1+\delta}{2} - \frac{p_2}{\mathbb{E}[V]}\right) + \frac{2\mathbb{E}[V]}{\beta+1} D_2^S$
(a_1, a_S)	$\frac{p_1}{\delta}\left(\frac{1+\delta}{2} - \frac{p_1}{\mathbb{E}[V]}\right) + \frac{\mathbb{E}[V]}{\delta(\beta+1)} \int_{(1-\delta)/2}^{\bar{\alpha}} \alpha\, d\alpha$	$\frac{\mathbb{E}[V]}{\delta(\beta+1)} \int_{(1-\delta)/2}^{\bar{\alpha}} (1-\alpha)\, d\alpha$
(a_2, a_S)	$\frac{\mathbb{E}[V]}{\delta(\beta+1)} \int_{\underline{\alpha}}^{(1-\delta)/2} \alpha\, d\alpha$	$\frac{p_2}{\delta}\left(\frac{1+\delta}{2} - \frac{p_2}{\mathbb{E}[V]}\right) + \frac{\mathbb{E}[V]}{\delta(\beta+1)} \int_{\underline{\alpha}}^{(1-\delta)/2} (1-\alpha)\, d\alpha$
(a_1)	p_1	0
(a_2)	0	p_2
(a_S)	$\frac{\mathbb{E}[V]}{\delta(\beta+1)} \int_{(1-\delta)/2}^{(1+\delta)/2} \alpha\, d\alpha$	$\frac{\mathbb{E}[V]}{\delta(\beta+1)} \int_{(1-\delta)/2}^{(1+\delta)/2} (1-\alpha)\, d\alpha$

of p_1 (see Figure 2). The revenue function is composed of the $\Pi_1(p_1; p_2)$s that correspond to each region (taken from Table 1). Thus, for a given p_2, firm 1's revenue function is continuous and piecewise concave and uniqueness of equilibrium cannot be guaranteed.

4.3. Equilibrium Analysis

Before finding the subgame perfect Nash equilibrium of the game, we discuss the equilibrium of a special case in which one of the firms is restricted to sell only on the spot and the other firm decides whether or not to sell in advance, where it is essentially a monopoly. Should it sell in advance? The literature on advance selling for a monopoly confirms that it should when $\delta = 0$. We find that this holds for all $\delta \in [0, 1]$: the monopolist always chooses to sell in advance. Lemma 2 characterizes the equilibrium for a general value of δ.

Lemma 2. *Let $\tilde{\delta} = \beta/(1+3\beta)$. If firm 2 sells only on the spot (by charging a high advance price, p_2), then* (i) *if $\delta < \tilde{\delta}$, the unique equilibrium is one in which firm 1 charges $p_1^{\text{all}} = (1-\delta)\mathbb{E}[V]/2$ in the advance period and sells to all consumers in advance*; (ii) *otherwise, the unique equilibrium is one in which firm 1 charges p_1^{part} in advance and advance sells only to consumers with $\alpha > p_1^{\text{part}}/\mathbb{E}[V]$, where*

$$p_1^{\text{part}} = \frac{(1+\delta)(1+\beta)\mathbb{E}[V]}{2(1+2\beta)}. \tag{5}$$

Lemma 2 suggests that there exists a $\tilde{\delta}$ so that the firm sells to all consumers in advance if $\delta < \tilde{\delta}$. Otherwise, the firm charges a relatively high advance price and sells to the more informed consumers, those with high values of α, in advance and to the less informed consumers on the spot. Regardless, at least in some capacity, advance selling is always beneficial to the monopolist.

Next, we turn to the construction that both firms can sell in advance, but assume that consumers have no information regarding their preferred firm in the advance period (i.e., $\delta = 0$). The equilibrium outcome is summarized in the following lemma.

Lemma 3. *When $\delta = 0$, the unique set of advance period prices is $p_1^* = p_2^* = 0$ and all consumers purchase in advance from one of the two firms. Any division of market demand between the two firms at these prices is an equilibrium.*

Lemma 3 demonstrates that advance selling always occurs in equilibrium when two firms are competing and consumers have no information in the advance period with respect to their eventual firm preference. However, while a monopolistic firm benefits from the time separation of purchase from consumption, which allows the monopolist to sell to a homogenous group of consumers, a price setting competitive firm does not. When the firms are identical, the only price equilibrium is one in which both firms charge an advance price of zero, all consumers purchase in advance, and both firms obtain zero revenues. Because the firms are identical, each firm has an incentive to undercut each other's advance price and obtain the entire market demand. As consumers are only interested in obtaining one unit, each firm knows that if it does not sell in advance, it gets no demand in the spot period. This leads to an intense Bertrand competition in the advance period, which eventually results in no revenue for the firms.[5] Thus, the benefit that advance selling had for a single firm is completely eliminated in this competitive case. If the firms were able to commit to sell only on the spot, they would obtain strictly positive profits. Thus, the possibility to advance sell makes both firms worse off. Limited capacity would mitigate the severity of this result, i.e., the equilibrium price would not fall to zero, but limited capacity does not change the fact that competition on homogeneous customers is intense and generally not beneficial to firms.

Next, we analyze the equilibrium of the general game. In this case, both firms are allowed to sell in advance and consumers are heterogeneous in the advance period, with degree of heterogeneity δ. We find that there are two possible symmetric price equilibria depending on the parameter conditions. Theorem 1 describes the equilibrium advance period prices and the corresponding consumer behavior.

Theorem 1. *Two symmetric price equilibria are possible*:

1. *For every β, there exists a $\delta_1(\beta)$, such that for every β and $\forall\, \delta \le \delta_1(\beta)$, the firms charge $p_1^l = p_2^l = \delta\,\mathbb{E}[V]$ in advance. All consumers purchase in advance and $\hat{\alpha} = 1/2$. Consumers with $\alpha > \hat{\alpha}$ buy in advance from firm 1 and consumers with $\alpha < \hat{\alpha}$ buy in advance from firm 2.*

2. *For every β, there exists a $\delta_2(\beta)$, such that for every β and $\forall\, \delta \ge \delta_2(\beta)$, the firms charge*

$$p_1^h = p_2^h = \frac{(1+\delta)(1+\beta)\mathbb{E}[V]}{2(1+2\beta)}.$$

Furthermore,

$$\bar{\alpha} = 1 - \underline{\alpha} = \frac{(1+\delta)(1+\beta)}{2(1+2\beta)} < 1.$$

Customers with $\alpha > \bar{\alpha}$ purchase in advance from firm 1, those with $\alpha < \underline{\alpha}$ purchase from firm 2, and the rest wait for the spot.

Theorem 1 demonstrates that there are two types of symmetric equilibria in this game. The first type of equilibrium (case 1 of Theorem 1) is one in which both firms charge a relatively low price that makes all consumers purchase in advance. Half of the consumers—those with high values of α—purchase in advance from firm 1 and the other half purchases in advance from firm 2. In the second type of equilibrium (case 2 of Theorem 1), the firms charge a higher price, which makes

only some consumers, those who are more informed, buy in advance. The other, relatively uninformed consumers, wait for the spot period to make their purchasing decisions.

In the low price equilibrium both firms charge $p_1^l = p_2^l = \delta\,\mathbb{E}[V]$. Each firm sells in advance to half of the consumers. Consumers with $\alpha > 1/2$ buy from firm 1 and consumers with $\alpha < 1/2$ buy from firm 2. This equilibrium occurs when customers are more a priori homogeneous (δ is low) and when the relative advantage of spot selling is low (β is high). In this case, in the advance period the firms engage in intense price competition as consumers are very price sensitive—they are relatively indifferent between the two firms, so price is low, because it is the key decider as to which firm to purchase from. Consequently, the firms capture the entire market in the advance period and their revenue is $\Pi_1 = \Pi_2 = \delta\,\mathbb{E}[V]/2$.

In the high price equilibrium, both firms charge $p_1^h = p_2^h = ((1+\delta)(1+\beta)/(2(1+2\beta)))\,\mathbb{E}[V]$ and sell to only a fraction of the consumers, the well-informed ones, in advance, while the relatively uninformed consumers wait for the spot period. This equilibrium occurs when δ is high and when β is low. When δ is high, consumers are more heterogeneous in the advance period. This decreases the firms' need to compete in advance to get demand—well-informed consumers will not purchase in advance from the firm for which they attach a low preference probability—and therefore the equilibrium prices are higher. When β is low, firms have more incentive to sell on the spot and, in fact, in this equilibrium the firms sell to some consumers on the spot. In this equilibrium, there does not exist a consumer who is indifferent between purchasing from either firm in the advance period and the advance period prices are equal to the monopolist's price (see Equation (5)). However, this should not be taken to mean that this equilibrium outcome is not due to competition: given that one firm sets a high price, p_j^h, to sell to informed consumers in advance, the other firm benefits from doing the same, even though, as is shown later (Section 4.4), in the majority of cases, both firms prefer that they both only sell on the spot.

Both the optimal advance prices and the resulting total expected profits increase in δ and decrease with β. Greater consumer heterogeneity in advance implies that some consumers become more informed, so the firms can charge a higher advance period price. As β decreases, firms have an incentive to sell more on the spot so they increase the advance price. This results in fewer consumers buying in the advance period, but at a higher price, and in more consumers who wait and purchase at v_h.

Note that since the firms' revenue functions are not quasiconcave, uniqueness of equilibrium cannot be guaranteed. In fact, the next theorem shows that under some parameter values, both equilibria exist.

Theorem 2. *$\delta_1(\beta) > \delta_2(\beta)\,\forall\,\beta$. Thus, a symmetric equilibrium always exists, but it is not necessarily unique*: *$\forall\,\delta \in [\delta_2(\beta), \delta_1(\beta)]$, both symmetric equilibria of Theorem 1 exist. Furthermore, in that range, the equilibrium where only part of the consumers purchase in advance* (*case* 2 *of Theorem* 1) *Pareto dominates the equilibrium in which all consumers purchase in advance* (*case* 1 *of Theorem* 1).

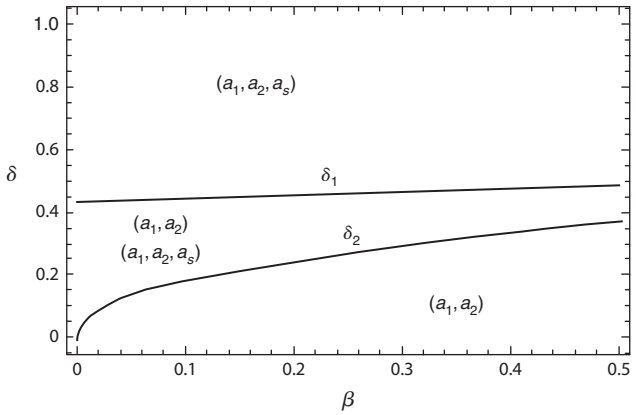
Figure 3 illustrates the ranges for which each of the two equilibria occurs on the (β, δ) parameter space. As shown in Theorem 2, a symmetric equilibrium always exists, but it is not necessarily unique. When δ is low (the bottom area) the unique equilibrium is such that the firms charge a low advance period price and all consumers purchase in advance (case 1 of Theorem 1). When δ is high (the upper area) the unique equilibrium has the firms charge a higher advance price and sell to some consumers in advance whereas some consumers wait for the spot (case 2 of Theorem 1). For midvalues of δ (the middle area between the two curves), both symmetric equilibria exist. Theorem 2 demonstrates that in this range, the higher price equilibrium (case 2) Pareto dominates the low price equilibrium (case 1).

Figure 3 suggests that competing firms prefer a market with a higher δ—with a high δ there are more consumers who are relatively certain of their preferences, which dampens competition between the firms, whereas with a low δ, consumers are uncertain of their preferences and competition is intense. The opposite holds for a monopolist, as confirmed by the next corollary.

Corollary 1. *As δ increases, the equilibrium revenue of the monopolist decreases, but the revenues of firms under competition increase.*

Hence, while a monopolist can use advance selling to profit from consumer homogeneity resulting from

Figure 3. Equilibrium Types on a (β, δ) Parameter Space



Notes. The tuple (a_1, a_2) corresponds to the equilibrium in which all customers purchase in advance (case 1 of Theorem 1) and (a_1, a_2, a_s) corresponds to the equilibrium in which some customers purchase in advance and others wait for the spot (case 1 of Theorem 1).

the time separation of purchase and consumption, this same homogeneity works against competing firms as it increases the intensity of price competition in the advance period.

4.4. The Choice to Sell in Advance

We know that a monopolist willingly chooses to sell in advance. Does the same hold for competing firms? We find that under most parameter values, it does not. Recall from Theorem 1 that some sort of advance selling always occurs in equilibrium. However, even though advance selling is always an equilibrium, in most cases, the firms would be better off if they could commit not to sell in advance.

Corollary 2. *Firms' revenues from selling only on the spot are given by*

$$\Pi_1 = \Pi_2 = \frac{v_h}{4} = \frac{\mathbb{E}[V]}{2(1+\beta)}.$$

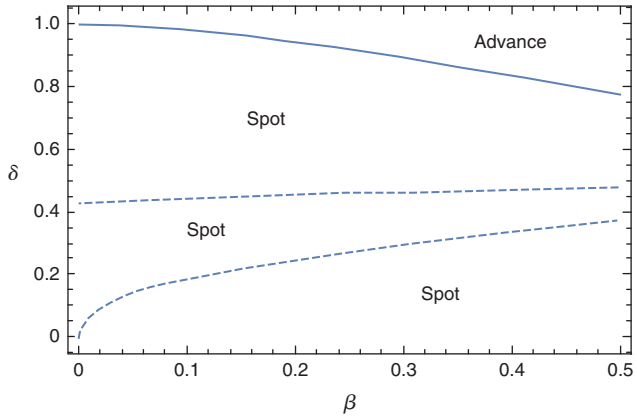
These revenues are strictly higher than the revenues obtained in the low price equilibrium range (*case* 1 *of Theorem* 1) *and are strictly higher than the high price equilibrium revenues* (*case* 2 *of Theorem* 1) *if*

$$\delta(\beta) < \frac{1+2\beta-\beta^2}{(1+\beta)^2}. \tag{6}$$

Corollary 2 demonstrates that advance selling is inferior in this model for most parameter values. If δ is low and consumers are rather homogeneous in advance, fierce competition in the advance period results in setting low prices and selling to all consumers, which clearly hurts profits. Even when customers are rather heterogeneous in advance and in equilibrium firms charge the monopoly price and sell only to the informed consumers in advance, advance selling may result in lower revenues compared to selling only on the spot. This is because under competition, when firms sell in advance to uninformed consumers, they lose the opportunity to sell to customers who purchased in advance from the other firm, but would have otherwise bought on the spot. Such a situation does not occur when one of the firms is restricted to sell on the spot, but happens under competition. Only when δ is high, so that there is little competition in advance (specifically, if condition (6) fails), advance selling results in higher profits compared to selling on the spot.

Hence, in most cases, the possibility of advance selling ends up hurting firms under competition. Firms would be better off if they were both able to commit to sell only on the spot. In these cases, the firms are in a prisoner's dilemma situation—even though the firms are better off selling only on the spot, they both are forced to sell in advance (given that the other firm sells in advance). Corollary 2 also demonstrates that

Figure 4. (Color online) Areas in the (β, δ) Parameter Space for Which the Revenue Obtained by Selling Only on the Spot Is Preferred to the Revenue Obtained in the Advance Selling Equilibrium



for high values of β and δ, both firms obtain higher revenues from advance selling, rather than spot selling. That is, under these parameter values, selling in advance (to highly informed consumers) is, in fact, a good pricing strategy for firms operating in a competitive environment, essentially because there is very little advance period competition when δ is large.

Figure 4 illustrates the range for which advance selling yields higher revenues for both firms (the area above the solid line) and the range where, if possible, the firms would benefit from committing to sell on the spot (the areas below the solid line). The latter decreases with an increase in β. When β decreases, the firms' benefit from spot selling increases and therefore firms increase the advance period price to make more consumers purchase in the spot period. In the limit, when $\beta = 0$, the price charged in advance is so high, that none of the consumers purchase in advance.

In sum, as long as advance-period heterogeneity is not too high, the possibility to advance sell hurts firms—firms would be better off if they could commit to sell only on the spot. Competitive firms can benefit from advance selling only if consumers are heterogeneous enough in the advance period, because in this case there is limited competition in the advance period—firms charge the monopoly price in the advance period, but are still able to sell to a large fraction of consumers on the spot at a high price.

5. Model II: Spot Period Competition

In the previous section, we showed that advance selling is in most cases an undesirable strategy when firms compete in the advance period. This occurs because firms who compete on a homogeneous market of consumers are pushed to decrease their prices, which makes selling to such consumers unprofitable. Thus, for the same reason that advance selling is attractive

for a monopoly, it is a problem for competitive firms. In this section, we eliminate advance period competition and investigate whether the benefits of advance selling survive spot period competition.

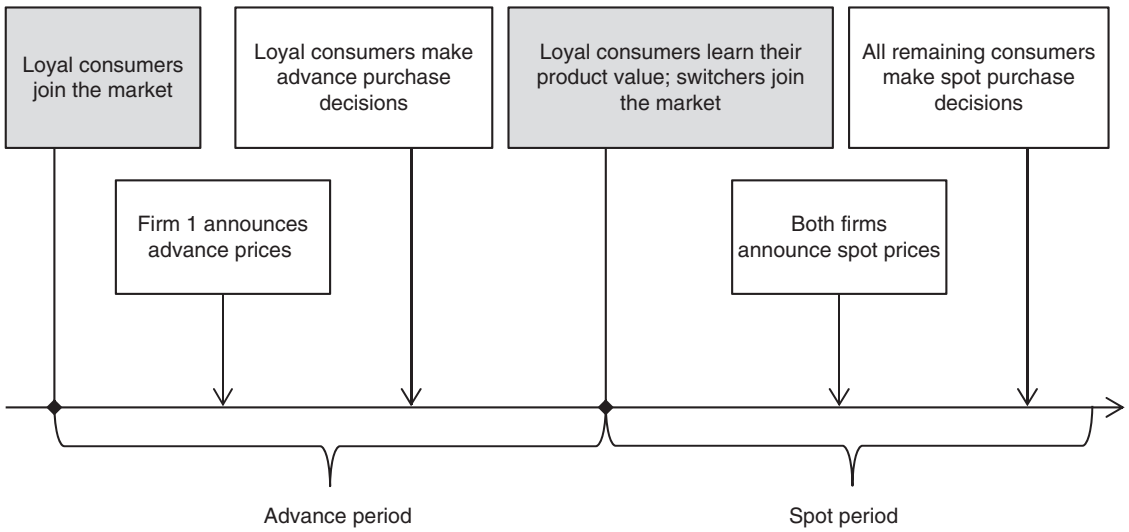
In this model, the market consists of two customer types, which taken together can be normalized to size 1 without loss of generality. The first type of consumers of size $\phi \in (0,1)$ arrives in the advance period. These consumers are loyal to firm 1 and never consider buying from firm 2. Loyal consumers consider whether to purchase from firm 1 in advance, to postpone their purchasing decision to the spot period, or to purchase nothing if the net value of doing so is negative (i.e., they have a "no-purchase" option). As in model I, these consumers are uncertain about their spot period value of the product. We assume that their value V is uniformly distributed between 0 and v and that their exact value is realized and observed in the spot period. The second type of consumers, of size $1-\phi$, arrive to the market only in the spot period. These consumers are switchers and are willing to purchase from either firm depending on prices and their preferences. To be specific, switchers are uniformly distributed along a single dimensional preference space of length v and incur a disutility of t per unit of distance from a firm. Thus, a switcher "located" at $x \in [0, v]$ prefers buying from firm 1 if $v - p_1 - tx \geq v - p_2 - t(v - x)$, otherwise firm 2 is preferred. Switchers also have a zero-value no-purchase option. As is later determined, the parameter t regulates the intensity of price competition between the two firms in the spot period. For the equilibrium to hold, switchers should have higher average values than loyals. This assumption may hold for sports and entertainment products where loyal consumers that are local may have a lower willingness to pay for a ticket compared to switchers who are in the area for a short period of time and have a rare opportunity to attend (e.g., anecdote 4 in Section 3). Another type of product that fits this setting is software, where loyal consumers are often those who buy the product for personal use and have a relatively low willingness to pay, whereas switchers are business consumers who have a higher willingness to pay, but evaluate the different products to determine the best fit.[6]

In this model, competition arises only in the spot period: firm 1 acts as a monopoly when serving its loyal consumers, but competes on prices to serve switchers and the loyals who waited for the spot period. Both firms set prices to maximize expected revenues. Firm 1 sets an advance period price to sell to loyal consumers and then both firms set spot period prices. We assume that firm 1 cannot price discriminate between switchers and loyal consumers. That is, firm 1 sets a single spot price at which both switchers and loyal customers can purchase. We look for a subgame perfect Nash equilibrium where firm 1 sets an advance period price and then both firms set spot period prices, loyal consumers choose whether to purchase in advance or wait for the spot, and switchers arriving on the spot decide whether to purchase and from which firm. We allow loyal consumers to adopt a mixed strategy—let γ be the probability that a loyal consumer waits for the spot period and let $1-\gamma$ be the probability that she purchases from firm 1 in advance. In equilibrium, γ determines the fraction of loyal consumers who decide to wait for the spot. Figure 5 displays the sequence of events for Model II.

5.1. The Monopoly Benchmark

To provide a benchmark, we first briefly discuss the equilibrium of the monopoly case, i.e., the model in which firm 2 does not exist but all other aspects of the

Figure 5. Sequence of Events in Model II



model remain. The equilibrium is summarized in the following lemma.

Lemma 4. *Let p_a be the advance period price and p_s be the spot period price in equilibrium. A monopolist sets $p_a = \max\{v(1-t^2)/2, 3v/8\}$ and $p_s = \max\{v(1-t), v/2\}$ and sells to all loyal consumers in advance, i.e., $\gamma^* = 0$. Furthermore, if $t \le 1/2$, all switchers purchase on the spot. Otherwise, only a fraction of switchers purchase.*

Lemma 4 demonstrates that advance selling is again desirable for a monopoly regardless of the parameter values—despite having to charge a lower price in advance, the monopolist prefers to sell to all of the loyal consumers in advance rather than to a portion of them in the spot period.

5.2. Spot Period Competition

With both firms 1 and 2 present in the spot period, the firms set spot period prices to compete for switchers. Loyal consumers anticipate spot period prices when deciding whether to buy in advance or to wait. We first analyze the spot period subgame and then solve for the equilibrium of the entire game.

In the spot period subgame, the market consists of a mass of $\phi\gamma$ consumers loyal to firm 1 and $1-\phi$ switchers. Loyal customers have value heterogeneity and switchers are heterogeneous with respect to firm preferences, but all customers know their preference location and value for the product.

Suppose that the firms set p_1 and p_2 as their spot prices. A loyal consumer with realized value, v, purchases from firm 1 if $v \ge p_1$. A switcher with realized preference x may purchase from either firm. She may do one of the following:

1. Buy on the spot from firm 1 and get utility $v - p_1 - tx$.
2. Buy on the spot from firm 2 and get utility $v - p_2 - t(v-x)$.
3. Do not buy, which yields a utility of 0.

Combining switchers' purchasing decisions and using an argument similar to Lemma 1, all switchers with

$$x \le \min\left\{\frac{v-p_1}{t}, \frac{p_2-p_1+vt}{2t}\right\} \tag{7}$$

buy from firm 1 and all switchers with

$$x \ge \max\left\{\frac{v-p_2}{t}, \frac{p_2-p_1+vt}{2t}\right\} \tag{8}$$

buy from firm 2. If $p_1 + p_2 \le (2-t)v$, all switchers buy. Switchers with $x > \hat{x}$ buy from firm 1 and those with $x < \hat{x}$ buy from firm 2, where

$$\hat{x} = \frac{p_2 - p_1 + vt}{2t}.$$

However, if $p_1 + p_2 > (2-t)v$, then switchers with $x \in (\underline{x}, \bar{x})$ do not buy from either firm, where

$$\underline{x} = \frac{v-p_1}{t}$$

and

$$\bar{x} = \frac{p_2 - v(1-t)}{t}.$$

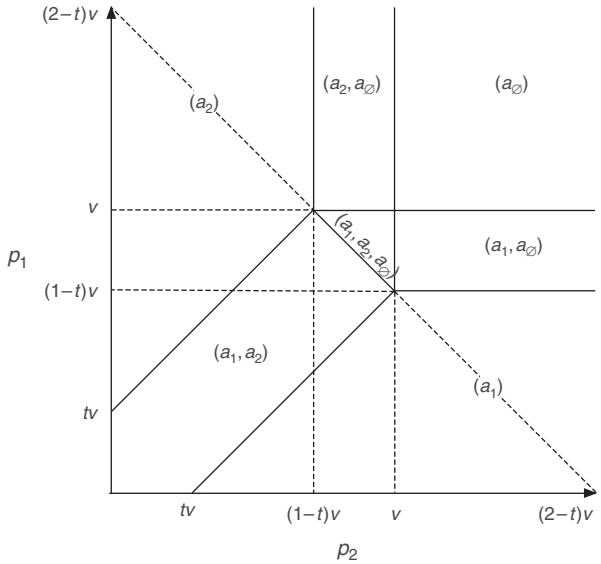
Figure 6 demonstrates the switchers' behavior on the spot-price space for $t \le 1/2$. The possible purchasing decisions are represented by $a_i, i \in \{1, 2, \varnothing\}$, where 1 denotes buying on the spot from firm 1, 2 denotes buying on the spot from firm 2, and $\varnothing$ denotes not buying. The same seven regions emerge when $t > 1/2$, and the only difference is that $(1-t)v < tv$.

For each set of spot prices (p_1, p_2), the firms can predict their spot period demand. Given that a fraction γ of loyals wait for the spot, the fraction of loyals who purchase from firm 1 on the spot is

$$D^l = \phi\gamma\frac{(v-p_1)^+}{v}.$$

Demand from switchers depends on (p_1, p_2) and can be inferred from switchers' behavior in Figure 6 and inequalities (7) and (8). If $p_1 + p_2 \le (2-t)v$, all switchers purchase on the spot, firm 1's demand from switchers is $D_1^s = (1-\phi)F(\hat{x})$ and firm 2's demand is $D_2^s = (1-\phi)\bar{F}(\hat{x})$. If, however, $p_1 + p_2 > (2-t)v$, then only a fraction of switchers purchase on the spot, and demand from the firms is $D_1^s = (1-\phi)\,\underline{x}/v$ and $D_2^s = (1-\phi)(1-\bar{x}/v)$. The firms' spot period revenues depend on both prices and are listed in Table 2. The notation for customer behavior is analogous to the one in Model I. The

Figure 6. Switchers' Behavior in a (p_1, p_2) Spot Price Space and $t \le 1/2$



Note. An a_i for a particular (p_1, p_2), denotes that there are switchers that follow purchasing strategy i, for that price tuple, $i \in \{1, 2, \varnothing\}$.

Table 2. Firms Revenues as a Function of Customers' Purchasing Behavior and Prices

	$\Pi_1(p_1;p_2)$	$\Pi_2(p_2;p_1)$
(a_1,a_2)	$p_1\left(\phi\gamma\frac{v-p_1}{v}+(1-\phi)\frac{1}{2}\left(1-\frac{p_1-p_2}{tv}\right)\right)$	$p_2(1-\phi)\frac{1}{2}\left(1-\frac{p_2-p_1}{tv}\right)$
$(a_1,a_2,a_\varnothing)$	$p_1\left(\phi\gamma\frac{v-p_1}{v}+(1-\phi)\frac{v-p_1}{tv}\right)$	$p_2(1-\phi)\frac{v-p_2}{tv}$
$(a_1,a_\varnothing)$	$p_1\left(\phi\gamma\frac{v-p_1}{v}+(1-\phi)\frac{v-p_1}{tv}\right)$	0
$(a_2,a_\varnothing)$	$p_1\left(\phi\gamma\frac{v-p_1}{v}\right)$	$p_2(1-\phi)\frac{v-p_2}{tv}$
(a_1)	$p_1\left(\phi\gamma\frac{v-p_1}{v}+(1-\phi)\right)$	0
(a_2)	$p_1\left(\phi\gamma\frac{v-p_1}{v}\right)$	$p_2(1-\phi)$
$(a_\varnothing)$	0	0

revenue functions are continuous and piecewise concave in a firm's own price.

The next lemma establishes the equilibrium of the spot period subgame.

Lemma 5. *There exist two thresholds $\tilde{t}(\phi,\gamma)\le 1/6$ and $\bar{t}(\phi,\gamma)>2/3$, such that for every $t\in[\tilde{t}(\phi,\gamma),\bar{t}(\phi,\gamma)]$, there exists a unique spot period price equilibrium, $p_i^s(\gamma)$, $i=1,2$, which is given by*

$$p_1^s=\frac{(3-(3-4\gamma)\phi)tv}{3-\phi(3-8\gamma t)};\quad p_2^s=\frac{(3-(3-\gamma(2+4t))\phi)tv}{3-\phi(3-8\gamma t)}. \tag{9}$$

Otherwise, if $t>\bar{t}(\phi,\gamma)$, then there exist infinitely many spot period price equilibria that satisfy

$$\begin{cases} p_1+p_2=(2-t)v \\ \max\{v/2,\bar{p}_2\}\le p_2\le\min\{2v/3,3v/2-tv\}, \end{cases}$$

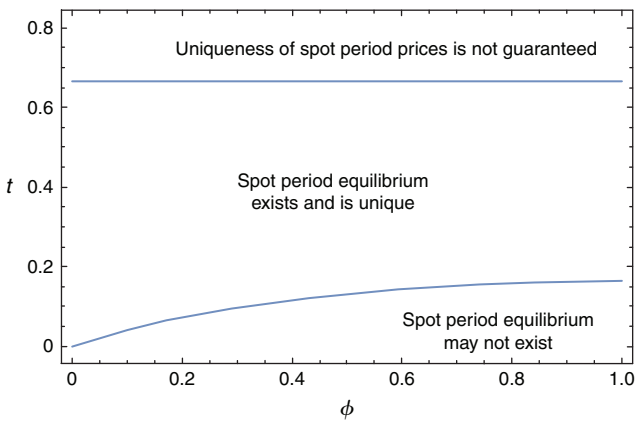
where

$$\bar{p}_2=\frac{v((4-3t)(1-\phi)+\gamma\phi t(6-4t))}{3(1-\phi)+4\gamma\phi t},$$

and if $t<\tilde{t}(\phi,\gamma)$, there does not exist a spot period equilibrium. Moreover, $\tilde{t}(\phi,\gamma)$ and $\bar{t}(\phi,\gamma)$ are increasing functions of γ and ϕ.

Lemma 5 provides a range of the preference parameter t that guarantees the existence and uniqueness of a spot price equilibrium. If $t>\bar{t}(\phi,\gamma)$, there may exist infinitely many spot period equilibria. To explain, in this case a firm can increase its share of switchers only by choosing a large price reduction. As a result, in equilibrium, each firm charges a spot price to defend its share of the market but is unwilling to fight further to increase its share. The region of multiple equilibria is maximized when $\gamma\to 0$, in which case $\lim_{\gamma\to 0}\bar{t}(\phi,\gamma)=2/3$. Alternatively, if $t<\tilde{t}(\phi,\gamma)$, there does not exist a spot period equilibrium. The nonexistence stems from having two distinct segments of consumers, loyals and switchers, which may result in a discontinuity in firm 1's best response function. If the fraction of loyal consumers remaining in the spot period, $\phi\gamma$, is large and the competition on switchers is high (t is small), firm 1 may respond to firm 2's low price by increasing its price to $v/2$ to sell only to its loyal consumers. That encourages firm 2 to raise its own price, which motivates firm 1 to lower its price and sell to some switchers as well, resulting in cyclical behavior without convergence. As $\tilde{t}(\phi,\gamma)$ increases in γ, the region of nonexistence is maximized when $\gamma=1$. Figure 7 illustrates the minimal region for which there exists a unique spot period equilibrium (given in Lemma 5).

Figure 7. (Color online) Range in the (ϕ,t) Space Where There Exists a Unique Spot Period Price Equilibrium for All Values of γ



For the remaining analysis, we restrict attention to the parameter range that guarantees that the spot period price equilibrium exists and is unique for any purchase probability, γ, i.e., we focus on $t \in [\tilde{t}(\phi,1),2/3]$. Observe that in this range firm 1's spot period price, $p_1^s(\gamma;t)$ is monotone in γ—it is increasing in γ when $t \le 1/2$ and is decreasing otherwise. To explain, there are two effects that contribute to firm 1's choice of a spot period price. The first effect is the price response to cater to both loyals and switchers. Firm 1 sets a spot price based on the size and average valuations of the two populations. Because average valuations of loyals is lower than that of switchers, a larger fraction of loyal consumers in the spot decreases p_1^s. This effect uniquely contributes to the spot period price for the monopolist (see proof of Lemma 4) and would determine the spot period price for firm 1 if it were naïve. However, a strategic firm considers a second effect due to competition. Firm 1 responds to firm 2's spot period price. Adding more loyals to the spot period reduces firm 1's incentive to undercut its rival to fight for switchers and results in an increase in p_1^s. Overall, which effect dominates depends on the level of competitiveness in the spot period that is determined by the parameter t. Small t values imply that the switcher market is more competitive and therefore adding more loyals to the spot period increases spot period prices.

Next, we analyze the advance-period equilibrium price. Loyal consumers who consider purchasing in advance can anticipate the equilibrium spot price, given firm 1's advance price and their expectation of the fraction of loyal consumers who purchase in advance. Their expected utility from waiting for the spot is

$$\begin{aligned}\mathbb{E}[U^s(p_1^s(\gamma;\phi,t))] &= \mathbb{P}\{V \ge p_1^s\}(\mathbb{E}[V \mid V \ge p_1^s] - p_1^s)\\ &= \frac{(v-p_1^s)^2}{2v},\end{aligned}$$

where p_1^s is a shorthand notation for firm 1's spot period equilibrium price (given in (9)) and is a function of the fraction of loyal consumers that remain in the market, γ. Clearly, the lower the spot period price, the more likely it is that a loyal consumer waits for the spot period. To make a fraction $(1-\gamma)$ of consumers buy in advance, firm 1 must charge an advance price that is sufficiently low and one that decreases if the spot price decreases

$$\begin{aligned}p_1^a(\gamma;\phi,t) &= \mathbb{E}[V] - \mathbb{E}[U^s(p_1^s(\gamma;\phi,t))]\\ &= \frac{v}{2} - \frac{(v-p_1^s)^2}{2v}. \qquad (10)\end{aligned}$$

The firm can control the fraction of consumers who wait for the spot period by changing the advance period price. Observe that $p_1^a(\gamma;t)$ is monotone in γ—it is increasing in γ when $t \le 1/2$ and is decreasing otherwise. Therefore, the firm's optimization problem may be solved in terms of γ instead of the advance period price, p_1^a. Firm 1's total revenue function as a function of γ is given by

$$\Pi_1^T(\gamma) = \phi(1-\gamma)p_1^a(\gamma) + \Pi_1^s(p_1^s;p_2^s),$$

where p_1^s and p_2^s are functions of γ and are given in Lemma 5 and $p_1^a(\gamma)$ is given in (10). Theorem 3 characterizes the fraction of loyal consumers who wait for the spot period in equilibrium, γ^*.

Theorem 3. *The equilibrium fraction of loyal consumers who wait for the spot depends on t and ϕ and is given by*

$$\gamma^* = \begin{cases} 1 & t \le 2/7,\ \phi \le \hat{\phi} \\ \hat{\gamma} & t \le 2/7,\ \phi > \hat{\phi} \text{ or } 2/7 < t \le 1/2,\ \phi > \bar{\phi} \\ 0 & \text{otherwise,} \end{cases}$$

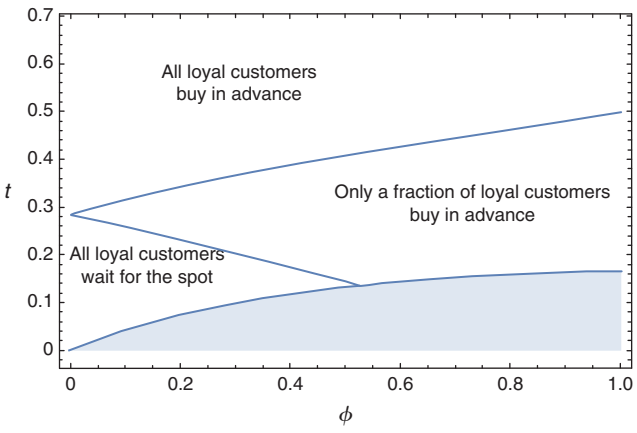
where $\hat{\gamma}$ is the unique solution to $d\Pi_1^T(\gamma)/d\gamma = 0$,

$$\hat{\phi} = (3(2-5t+4t^2) - 2t\sqrt{3(19-44t+12t^2)})/(6-9t-8t^2),$$

and $\bar{\phi} = (-2+7t)/(6-17t+16t^2)$.

Figure 8 illustrates the types of equilibria resulting in the game on the (t,ϕ) parameter space according to Theorem 3. Observe that low values of t yield an equilibrium with no advance sales. In this range, the spot period competition is very high. Intense spot period competition results in low spot prices, which make it unprofitable for firm 1 to sell in advance as loyal consumers anticipate a low future price. This is in contrast to the monopoly benchmark where the monopolist always benefits from selling to all loyal consumers in advance (irrespective of the level of t). Note that from a modeling perspective, the only departure from the monopoly benchmark is that the general model includes a second firm that competes for switchers on the spot. Firm 1 is a monopoly in advance and loyal consumers only consider purchasing from it in either

Figure 8. (Color online) Equilibrium Outcomes on the (t,ϕ) Parameter Space



period. Still, advance selling becomes unprofitable for low levels of t because of the low spot period prices induced by the competition on switchers. By contrast, high values of t result in an equilibrium where the firm sells to all loyal consumers in advance. In this case, advance selling is profitable even under spot period competition. However, this occurs when t is high, so switchers are more differentiated and the level of competition between the two firms is low, resulting in high spot period prices. Anticipating a high spot price, loyal consumers are willing to purchase in advance at a relatively high price. While the firms still compete in the spot period, when the level of competition is low, the result mimics the advance selling result of a monopolist. Finally, as the level of t increases from low to high, the fraction of consumers who purchase in advance gradually increases.

As Figure 8 illustrates, when a firm offers a product in both periods, but engages in price competition in the spot period, advance selling does not always occur in equilibrium, even when a firm is a monopoly in advance. Competition in the spot period may be sufficiently fierce, yielding low spot period prices, so that the firm cannot offer a profitable advance period price that attracts all loyal customers. Therefore, a monopolist that is strategic and anticipates spot period competition optimally chooses to limit advance selling or eliminates it altogether. By contrast, firms who do not experience competition always benefit from selling to all loyal consumers in advance.

6. Conclusion

It has been shown that a monopolist can benefit from advance selling because consumers are more homogeneous in the advance period than in the spot period. The monopolist must give an advance period discount, but because the monopolist is expected to charge a high spot period price, consumers choose to purchase in advance. There are two reasons why this logic does not carry over well into a competitive setting. First, as consumers are more homogeneous in the advance period, they may choose which firm to purchase from in advance primarily based on price (model 1). The resulting price competition lowers the advance period prices and therefore the attractiveness of selling in advance. Model 1 further demonstrates that in most cases advance selling occurs in equilibrium (at least to some degree), but the possibility of advance selling hurts competitive firms. In most of these cases, firms would be better off if they were able to commit not to advance sell. In such cases, advance selling may be better described as a competitive necessity rather than as an advantageous tool to raise profits. Second, competition in the spot period is likely to reduce the spot period price (model 2), which means that the firm must further discount the advance period price—consumers will not purchase in advance, unsure of their valuation, if they can anticipate a low price in the spot period, when they know they will learn their valuation. A rational monopolist that foresees this future competition chooses to either limit the practice of advance selling or abandon it altogether. Overall, we conclude that advance selling is less beneficial, and may even be harmful, under competition.

Acknowledgments

The authors thank the editor, associate editor, and referees for many useful suggestions that helped improve the paper.

Appendix. Proofs

Proof of Lemma 1. Let p_1 and p_2 be the firms' advance period prices. In equilibrium, v_h is the spot price charged by the two firms. The surplus of a customer with α who purchases in advance from firm 1 is $\alpha\,\mathbb{E}[V]-p_1$. If the same customer purchases in advance from firm 2, her expected surplus is $(1-\alpha)\mathbb{E}[V]-p_2$. Finally, if the customer waits for the spot, her expected surplus is 0. A customer purchases in advance from firm 1 if and only if

$$\alpha\,\mathbb{E}[V]-p_1 \geq \max\{(1-\alpha)\mathbb{E}[V]-p_2, 0\}. \tag{11}$$

Suppose there exists an $\bar{\alpha}$ such that $\bar{\alpha}\,\mathbb{E}[V]-p_1=\max\{(1-\bar{\alpha})\cdot \mathbb{E}[V]-p_2, 0\}$. Since the left-hand side increases with α and the right-hand side decreases with α, such $\bar{\alpha}$ is unique. Furthermore, condition (11) holds $\forall\,\alpha \geq \bar{\alpha}$ and fails otherwise. An analogous argument follows for purchasing in advance from firm 2. □

Proof of Lemma 2. Without loss of generality, let firm 2 be the firm restricted to selling only on the spot. (That is, firm 2 charges a high enough advance price, $p_2 > (1+\delta)\mathbb{E}[V]/2$, so that consumers do not purchase from it in advance.) Given this p_2, firm 1's revenue function is

$$\Pi_1(p_1;p_2)=\begin{cases} p_1, & p_1 \in [0,(1-\delta)\mathbb{E}[V]/2] \\ \dfrac{p_1}{\delta}\left(\dfrac{1+\delta}{2}-\bar{\alpha}\right)+\dfrac{\mathbb{E}[V]}{\delta(1+\beta)}\displaystyle\int_{(1-\delta)/2}^{\bar{\alpha}} \alpha\,d\alpha, & \\ \qquad p_1 \in ((1-\delta)\mathbb{E}[V]/2,(1+\delta)\mathbb{E}[V]/2] & \\ \dfrac{\mathbb{E}[V]}{\delta(1+\beta)}\displaystyle\int_{(1-\delta)/2}^{(1+\delta)/2} \alpha\,d\alpha=\dfrac{v_h}{4}, & \\ \qquad p_1 > (1+\delta)\mathbb{E}[V]/2. & \end{cases}$$

Observe that $\Pi_1(p_1;p_2)$ is continuous and is composed of three parts: (1) a linearly increasing function with slope 1; (2) a strictly concave function ($d\Pi_1^2/d^2p_1 = (1/(1+\beta)-2)/(\delta\,\mathbb{E}[V]) < 0$); and (3) a constant function. Thus, the optimal price, p_1^*, must fall in the range $[((1-\delta)/2)\mathbb{E}[V], ((1+\delta)/2)\mathbb{E}[V]]$. If $p_1^* = ((1+\delta)/2)\mathbb{E}[V]$, then there are infinitely many prices that result in the same optimal revenue. Otherwise, in that range both existence (from the maximum theorem) and uniqueness ($\Pi_1(p_1;p_2)$ is strictly concave in this range) is guaranteed. Differentiating the second segment of $\Pi_1(p_1;p_2)$, we get

$$\frac{d\Pi_1(p_1;p_2)}{dp_1}=\frac{p_1}{\delta\,\mathbb{E}[V]}\left(\frac{1}{1+\beta}-2\right)+\frac{1+\delta}{2\delta}.$$

Equating to zero and rearranging, we get

$$p_1^* = \frac{(1+\delta)(1+\beta)\mathbb{E}[V]}{2(1+2\beta)}. \tag{12}$$

The price given in (12) is the optimal advance price if it belongs to $[((1-\delta)/2)\cdot\mathbb{E}[V], ((1+\delta)/2)\mathbb{E}[V]]$. Otherwise, the optimal price is a corner solution. To see that $p_1^* \neq ((1+\delta)/2)\mathbb{E}[V]$, note that

$$\left.\frac{d\Pi_1(p_1;p_2)}{dp_1}\right|_{p_1=((1+\delta)/2)\mathbb{E}[V]} = -\frac{1+\delta}{2\delta}\frac{\beta}{1+\beta} < 0.$$

Applying some algebra to compare (12) to $((1-\delta)/2)\mathbb{E}[V]$, the result follows. □

Proof of Lemma 3. Let $p_k > 0$ be the advance period price charged by firm k, $k \in \{1,2\}$. Because customers only buy once and the firms are a priori identical, if firm i undercuts firm j's advance price by charging $p_i = p_j - \epsilon$, it gets all market demand and firm j gets no demand. The proof of this lemma is therefore analogous to the well-known proof of the Bertrand equilibrium for a zero marginal cost and will not be repeated. Refer to Kreps (1990, pp. 330–335), for a complete proof of the Bertrand result. □

Proof of Theorem 1. It follows from the profit functions in Table 1 that there are two candidates for a symmetric price equilibrium. The first candidate is a set of prices that results in all customers buying in advance (falls in the (a_1,a_2) range). The second candidate is a set of prices that results in only a fraction of consumers buying in advance (falls in the (a_1,a_2,a_s) range). We check under which conditions these prices are sustainable in equilibrium. In checking for profitable deviations, we focus on firm 1. The behavior of firm 2 is identical because of the symmetry of the game.

(i) The (a_1,a_2) range: Given p_2, in this range, $\Pi_1(p_1;p_2) = (p_1/(2\delta))(\delta - (p_1-p_2)/\mathbb{E}[V])$, which is strictly concave and maximized at $p_1(p_2) = (\delta\mathbb{E}[V]+p_2)/2$. Symmetry and the concavity of the profit functions in this range imply that the only interior candidate in this range is

$$p_1^l = p_2^l = \delta\mathbb{E}[V].$$

To show that it is an equilibrium, we check for which parameter values these prices fall in the (a_1,a_2) range and whether there are no profitable price deviations. If $\delta > 1/2$, the prices (p_1^l,p_2^l) fall outside the (a_1,a_2) range and therefore cannot be an equilibrium. Otherwise, (p_1^l,p_2^l) is in the range. If $\delta \le 1/3$, firm 1 has no profitable deviations and thus p_1^l is the optimal price in the (a_1,a_2) range. To see this, note that choosing any price outside this range, i.e., setting $p_1 > 2\delta\mathbb{E}[V]$ results in all customers buying in advance from firm 2 (range (a_2)), which is clearly not profitable. Finally, if $1/3 < \delta \le 1/2$, a price increase may result in some customers waiting for the spot (the (a_1,a_2,a_s) range). Increasing the price further results in no customers buying in advance from firm 1 (the (a_2,a_s) range). Can an increase in p_1 be profitable? The profit function in the (a_1,a_2,a_s) range is given by

$$\Pi_1(p_1;p_2) = \frac{p_1}{\delta}\left(\frac{1+\delta}{2} - \frac{p_1}{\mathbb{E}[V]}\right) + \frac{2\mathbb{E}[V]}{\beta+1}D_1^S. \tag{13}$$

Taking the first order conditions, we get that the function is maximized at p_1^h. If $p_1^h < \mathbb{E}[V] - p_2^l = (1-\delta)\mathbb{E}[V]$, then from continuity and the fact that the function is piecewise concave, it follows that (p_1^l,p_2^l) is an equilibrium. If, however, $p_1^h \in ((1-\delta)\mathbb{E}[V], (1+\delta)\mathbb{E}[V]/2]$, i.e., it falls in the (a_1,a_2,a_s) range, then comparing between the two profit functions, we get that deviating to p_1^h is profitable if $\delta_1 < \delta \le 1/2$, where

$$\delta_1 = \frac{5+10\beta+\beta^2-2(1-\beta)\sqrt{(1+\beta)(1+2\beta)}}{7+18\beta+7\beta^2}.$$

Finally, $p_1^h \not> (1+\delta)\mathbb{E}[V]/2$, so deviating to the (a_2,a_s) range cannot be profitable. Combining the conditions for deviation, we find that (p_1^l,p_2^l) is an equilibrium if and only if $\delta \le \delta_1$.

(ii) The (a_1,a_2,a_s) range: Given p_2, in this range, the profit function of firm 1 is given by Equation (13), which is strictly concave and maximized at p_1^h. This price is independent of p_2. For (p_1^h,p_2^h) to be an equilibrium, it must fall in the (a_1,a_2,a_s) range, i.e., we must have $p_1^h \in [\mathbb{E}[V]-p_2^h, (1+\delta)\mathbb{E}[V]/2]$, which happens $\forall\,\delta \ge \beta/(1+\beta)$. Therefore, (p_1^h,p_2^h) is not an equilibrium if $\delta < \beta/(1+\beta)$. Next, we check whether it is worthwhile to deviate from this price. Charging a price $p > p_1^h$ is surely not profitable, because the profit function is constant at range (a_2,a_s). So it remains to check whether lowering the price and deviating to ranges (a_1,a_2) or (a_1) is profitable. Deviation to the (a_1,a_2) range: the profit function in the (a_1,a_2) range is maximized at $p_1(p_2) = (\delta\mathbb{E}[V]+p_2)/2$. A deviation will be profitable, if $p_1(p_2 = p_2^h) \in (\max\{0, p_2^h - \delta\mathbb{E}[V]\}, \mathbb{E}[V]-p_2^h)$ and the profit from deviating is higher. Deviation to the (a_1) range is profitable if $p_1(p_2 = p_2^h) < \max\{0, p_2^h - \delta\mathbb{E}[V]\}$ and the profit from deviating to $p_1 = p_2^h - \delta\mathbb{E}[V]$ is higher. Combining the conditions, we get that (p_1^h,p_2^h) is an equilibrium if $\delta \ge \delta_2(\beta)$, where

$$\delta_2(\beta) = \begin{cases} \delta_3 & \beta > \bar{\beta} \\ \delta_4 & \text{otherwise,} \end{cases}$$

where $\bar{\beta}$ is the unique solution to the cubic equation $1-8\beta-31\beta^2+14\beta^3 = 0$ in the range $\beta \in [0,1/2]$ and is approximately equal to $\bar{\beta} \approx 0.093$,

$$\delta_3 = \frac{5+16\beta+3\beta^2-4(1-\beta)(1+2\beta)\sqrt{(1+2\beta)/(1+\beta)}}{9+22\beta+17\beta^2},$$

and

$$\delta_4 = \frac{\beta^2+(1+2\beta)\sqrt{\beta(2+7\beta-3\beta^2)/(1+\beta)}}{2+11\beta+13\beta^2}.$$

Finally, it remains to show that the prices $p_1 = p_2 = \mathbb{E}[V]/2$ and $p_1 = p_2 = (1+\delta)\mathbb{E}[V]/2$, i.e., prices in the boundaries, cannot be an equilibrium. For $p_1 = p_2 = \mathbb{E}[V]/2$ to be an equilibrium, we must have that $\delta > 1/2$ and that $p_1^h < \mathbb{E}[V]/2$. These two conditions cannot be satisfied together. For $p_1 = p_2 = (1+\delta)\mathbb{E}[V]/2$ to be an equilibrium, $p_1^h > (1+\delta)\mathbb{E}[V]/2$. This never holds, because $\beta \ge 0$. □

Proof of Theorem 2. Multiplicity of equilibria: it is sufficient to show that $\delta_1(\beta) > \delta_2(\beta)\ \forall\,\beta$. Since $\delta_3 \ge \delta_4\ \forall\,\beta \in [0,1/2]$, it is sufficient to show that $\delta_1(\beta) > \delta_3(\beta)\ \forall\,\beta$. Differentiating δ_k with respect to β, we get that $\delta_k'(\beta) > 0\ \forall\,k \in \{1,3\}$ for $\beta \in [0,1/2]$. Next, $\delta_1(\beta=0) = 3/7$ and $\delta_3(\beta=1/2) < 3/7$. This implies that an equilibrium always exists, but it is not necessarily unique: $\forall\,\delta$: $\delta \in [\delta_2,\delta_1]$, both (p_1^l,p_2^l) and (p_1^h,p_2^h) are sustainable.

Pareto dominance: an equilibrium is Pareto dominant if it is Pareto superior to all other equilibria in the game. To show

that the price pair (p_1^h, p_2^h) Pareto dominates the price pair (p_1^l, p_2^l), we need to show that $\Pi_1^*(p_1^h; p_2^h) > \Pi_1^*(p_1^l; p_2^l)$ in the range $\delta \in [\delta_2(\beta), \delta_1(\beta)]$, where

$$\Pi_1^*(p_1^h; p_2^h) = \frac{p_1^h}{\delta}\left(\frac{1+\delta}{2} - \frac{p_1^h}{\mathbb{E}[V]}\right) + \frac{\mathbb{E}[V]}{\beta+1}\int_{1-(p_2^h/\mathbb{E}[V])}^{(p_1^h/\mathbb{E}[V])} \alpha\, d\alpha$$
$$= \frac{\mathbb{E}[V]}{2\delta}\left(\frac{(1+\delta)^2(1+\beta)\beta}{2(1+2\beta)^2} + \frac{1+\delta}{1+2\delta} - \frac{1}{1+\beta}\right),$$

and $\Pi_1^*(p_1^l; p_2^l) = \delta\,\mathbb{E}[V]/2$. Comparing the profit functions, we get that $\Pi_1^*(p_1^h; p_2^h) > \Pi_1^*(p_1^l; p_2^l)\ \forall\ \delta(\beta) \in (\delta^-(\beta), \delta^+(\beta))$, where

$$\delta^{\pm}(\beta) = \frac{1+3\beta+\beta^2 \pm (1+2\beta)\sqrt{1-2\beta(1-\beta)/(1+\beta)}}{2+7\beta+7\beta^2}.$$

To prove Pareto dominance, we need to show that $\delta^-(\beta) \le \delta_4(\beta)$ and that $\delta_1(\beta) < \delta^+(\beta)\ \forall\ \beta$: We have $\delta^{-'}(\beta) > 0$ and $\delta_4'(\beta) > 0\ \forall\ \beta \in [0, 1/2]$. Furthermore, let $\hat{\beta}$ be the solution to the equation $\delta^-(\hat{\beta}) - \delta_4(\hat{\beta}) = 0$. Algebra reveals that $\hat{\beta} = 0$ is the unique solution to the equation. Thus, to show that $\delta^-(\beta) \le \delta_4(\beta)$, it is enough to find one $\beta \ne 0$, which satisfies the inequality. Take $\beta = 1/2$. $\delta^-(\beta = 1/2) < \delta_4(\beta = 1/2)$ and the result follows. Furthermore, since $\delta^{+'}(\beta) < 0$ and $\delta_1'(\beta) > 0$, it is enough to show that $\delta_1(\beta = 1/2) < \delta^+(\beta = 1/2)$. Plugging $\beta = 1/2$ in, we get the desired result. (Symmetry implies that $\Pi_2^*(p_1^h; p_2^h) > \Pi_2^*(p_1^l; p_2^l)$ as well.) □

Proof of Lemma 4. The firm's spot period revenue function is

$$\Pi_s = p\left(\phi\gamma\frac{v-p}{v} + (1-\phi)\min\left\{\frac{v-p}{tv}, 1\right\}\right),$$

which is piecewise concave and maximized at

$$p_s = \begin{cases} v/2 & t \ge 1/2 \\ \dfrac{v(1-\phi(1-\gamma))}{4\gamma\phi} & t < 1/2,\ \phi > \dfrac{1}{2(1-t)},\ \gamma > \dfrac{1-\phi}{\phi(1-2t)} \\ v(1-t) & \text{otherwise.} \end{cases}$$

In advance, loyals anticipate the spot period price, p_s. Their expected utility from waiting for the spot is $(\mathbb{E}[V \mid V \ge p_s] - p_s)\bar{F}(p_s)$. Therefore, the best advance period price to charge to make loyal consumers purchase in advance is $p_a = \mathbb{E}[V] - (\mathbb{E}[V \mid V \ge p_s] - p_s)\bar{F}(p_s) = v/2 - (v-p_s)^2/(2v)$ and the total revenue is $\phi(1-\gamma)p_a + \Pi_s^*$. If $t > 1/2$, $p_a = 3v/8$ and

$$\Pi_s = \frac{(1-\phi(1-\gamma t))v}{4t}.$$

The total revenue function is

$$\Pi = \frac{(2-\phi(2-(3-\gamma)t))v}{8t}.$$

Differentiating the revenue function with respect to γ, we find that $d\Pi/d\gamma = -\phi v/8 < 0$ and therefore $\gamma^* = 0$ and selling to all loyals in advance is optimal. For the parameter values where $p_s = v(1-t)$, $p_a = v(1-t^2)/2$ and $\Pi_s = v(1-t)(1-\phi(1-\gamma t))$. The total revenue is

$$\Pi = \frac{1-t}{2}(2-(1+\gamma)(1-t)\phi)v.$$

Differentiating the revenue function with respect to γ, we find that $d\Pi/d\gamma = -\phi v(1-t)^2/2 < 0$ and therefore selling to all loyals in advance is optimal. Finally, for $t < 1/2$ and $\phi > 1/(2(1-t))$, the revenue function $\Pi(\gamma)$ is composed of two parts depending on γ. Let $\hat{\gamma} = (1-\phi)/\phi(1-2t)$. If $\gamma \le \hat{\gamma}$, we found that $d\Pi/d\gamma < 0$. If $\gamma > \hat{\gamma}$, the total revenue is

$$\Pi = \frac{(\gamma^2\phi(2+\phi) + \gamma(3-4\phi+\phi^2) - (1-\phi)^2 - \gamma^3\phi^2)v}{8\gamma^2\phi},$$

which is decreasing in γ for the relevant parameter range. Therefore, it is optimal to sell to all loyals in advance. □

Proof of Lemma 5. The revenue functions in Table 2 are piecewise concave and continuous. Differentiating each revenue function to find the maximum for each part of the function separately and checking whether the solution falls within its range, we get that the best response functions depend on the parameter values. Firm 1's best response function depends on the values of t, ϕ, and γ. If $t \le 1/2$ there are seven types of best response functions depending on the parameters: If $0 \le \phi \le 1/(1+\gamma)$ and $0 \le t \le t_1$ or $1/(1+\gamma) < \phi \le \phi_7$ and $t_6 < t \le t_1$ then

$$p_1(p_2) = \begin{cases} v/2 & 0 \le p_2 \le p_2^4 \\ -tv + p_2 & p_2^4 < p_2 \le v. \end{cases}$$

If $0 \le \phi \le \phi_7$ and $t_1 < t \le \min\{t_2, t_7\}$ or $\phi_7 < \phi \le 1$ and $t_6 < t \le t_7$ then

$$p_1(p_2) = \begin{cases} v/2 & 0 \le p_2 \le p_2^2 \\ \dfrac{(p_2+tv)(1-\phi)+2\gamma\phi tv}{2(1-\phi+2\gamma\phi t)} & p_2^2 < p_2 \le p_2^1 \\ -tv + p_2 & p_2^1 < p_2 \le v. \end{cases}$$

If $0 \le \phi \le \phi_6$ and $t_2 < t \le t_7$ then

$$p_1(p_2) = \begin{cases} \dfrac{(p_2+tv)(1-\phi)+2\gamma\phi tv}{2(1-\phi+2\gamma\phi t)} & 0 \le p_2 \le p_2^1 \\ -tv + p_2 & p_2^1 < p_2 \le v. \end{cases}$$

If $\max\{t_2, t_7\} < t \le 1/2$ then

$$p_1(p_2) = \begin{cases} \dfrac{(p_2+tv)(1-\phi)+2\gamma\phi tv}{2(1-\phi+2\gamma\phi t)} & 0 \le p_2 \le p_2^6 \\ (2-t)v - p_2 & p_2^6 < p_2 \le v. \end{cases}$$

If $\phi_6 < \phi \le 1$ and $t_7 < t \le t_2$ then

$$p_1(p_2) = \begin{cases} v/2 & 0 \le p_2 \le p_2^2 \\ \dfrac{(p_2+tv)(1-\phi)+2\gamma\phi tv}{2(1-\phi+2\gamma\phi t)} & p_2^2 < p_2 \le p_2^6 \\ (2-t)v - p_2 & p_2^6 < p_2 \le v. \end{cases}$$

If $1/(1+\gamma) < \phi \le 1$ and $0 < t \le \min\{t_1, t_6\}$ then

$$p_1(p_2) = \begin{cases} v/2 & 0 \le p_2 \le p_2^4 \\ -tv + p_2 & p_2^4 < p_2 \le p_2^5 \\ \dfrac{(1-\phi+\gamma\phi)v}{2\gamma\phi} & p_2^5 < p_2 \le v, \end{cases}$$

otherwise

$$p_1(p_2)=\begin{cases} v/2 & 0\le p_2\le p_2^2\\ \dfrac{(p_2+tv)(1-\phi)+2\gamma\phi tv}{2(1-\phi+2\gamma\phi t)} & p_2^2<p_2\le p_2^1\\ -tv+p_2 & p_2^1<p_2\le p_2^5\\ \dfrac{(1-\phi+\gamma\phi)v}{2\gamma\phi} & p_2^5<p_2\le v. \end{cases}$$

If $1/2<t\le 1$ then

$$p_1(p_2)=\begin{cases} \dfrac{(p_2+tv)(1-\phi)+2\gamma\phi tv}{2(1-\phi+2\gamma\phi t)} & 0\le p_2\le p_2^6\\ (2-t)v-p_2 & p_2^6<p_2\le 3v/2-tv\\ v/2 & 3v/2-tv<p_2\le v, \end{cases}$$

where

$$p_2^1=\frac{3tv(1-\phi)+2\gamma\phi tv(1+2t)}{1-\phi+4\gamma\phi t};$$

$$p_2^2=\frac{v\sqrt{2\gamma\phi t(1-\phi+2\gamma\phi t)}-tv(1-\phi)-2\gamma\phi tv}{1-\phi};$$

$$p_2^4=\frac{v(1-\phi+\gamma\phi(1+2t)+\sqrt{1-2\phi+2\gamma\phi+\phi^2-2\gamma\phi^2})}{2\gamma\phi};$$

$$p_2^5=\frac{v(1-\phi+\gamma\phi(1+2tv))}{2\gamma\phi};$$

$$p_2^6=\frac{v((4-3t)(1-\phi)+\gamma\phi t(6-4t))}{3(1-\phi)+4\gamma\phi t};$$

$$\phi_6=\frac{1-\gamma(1+\sqrt{5})}{1-2\gamma-4\gamma^2};\quad \phi_7=\frac{8-(5+3\sqrt{17})\gamma}{2(4-5\gamma-8\gamma^2)};$$

$$t_1=\frac{-1+\phi+(1-(1-\gamma)\phi)\sqrt{(1-\phi)/(1-(1-2\gamma)\phi)}}{4\gamma\phi};$$

$$t_2=\frac{2\gamma\phi}{1-\phi+4\gamma\phi};$$

$$t_6=\frac{-1+\phi+\gamma\phi}{2\gamma\phi};$$

$$t_7=\frac{-3+\phi(3+2\gamma+(\sqrt{9-2(9-2\gamma)\phi+(9-4\gamma+4\gamma^2)\phi^2}/\phi))}{8\gamma\phi}.$$

Firm 1's best response functions are continuous everywhere except at $p_2=p_2^2$ and $p_2=p_2^4$. Firm 2's best response function depends on the value of t. If $0\le t\le 1/3$,

$$p_2(p_1)=\begin{cases}\frac{1}{2}(tv+p_1) & p_1\le 3tv\\ -tv+p_1 & \text{otherwise.}\end{cases}$$

If $1/3<t\le 1/2$

$$p_2(p_1)=\begin{cases}\frac{1}{2}(tv+p_1) & p_1\le 4v/3-tv\\ (2-t)v-p_1 & \text{otherwise.}\end{cases}$$

If $1/2<t\le 1$

$$p_2(p_1)=\begin{cases}\frac{1}{2}(tv+p_1) & p_1\le 4v/3-tv\\ (2-t)v-p_1 & 4v/3-tv<p_1\le 3v/2-tv\\ v/2 & \text{otherwise.}\end{cases}$$

Combining the best response functions, we find that the discontinuity in the best response function of firm 1 results in an inexistence of equilibrium if $t<\tilde{t}(\phi,\gamma)$, where

$$\tilde{t}(\phi,\gamma)=(-9+3\phi(6+\gamma(-8+A))+\phi^2(-9-3\gamma(-8+A)+8\gamma^2(1+A)))\cdot(12\gamma\phi(3-(3-8\gamma)\phi))^{-1},$$

and

$$A=\frac{(3-(3-4\gamma)\phi)\sqrt{9-6(3-4\gamma)\phi+(9-24\gamma+4\gamma^2)\phi^2}}{\gamma\phi(3-(3-8\gamma)\phi)}.$$

The threshold $\tilde{t}(\phi,\gamma)$ is the t that solves $p_2^s=\tilde{p}_2$, where

$$\tilde{p}_2=\frac{v(\sqrt{2\gamma\phi t(1-\phi+2\gamma\phi t)}-t(1-\phi)-2\gamma\phi t)}{1-\phi}.$$

Differentiating $\tilde{t}(\phi,\gamma)$, we get that $\partial\tilde{t}(\phi,\gamma)/\partial\phi>0$ and $\partial\tilde{t}(\phi,\gamma)/\partial\gamma>0$. Then, there exists a threshold $\bar{t}(\phi,\gamma)$, so that if $t>\bar{t}(\phi,\gamma)$, where

$$\bar{t}(\phi,\gamma)=(-9(1-\phi)+10\gamma\phi+\sqrt{81(1-\phi)^2+108\gamma\phi(1-\phi)+100\gamma^2\phi^2})\cdot(24\gamma\phi)^{-1},$$

then there exist infinitely many spot period price equilibria. Note that $\bar{t}(\phi,\gamma)$ is the t that solves $p_2^s=\bar{p}_2$, where

$$\bar{p}_2=\frac{v((4-3t)(1-\phi)+\gamma\phi t(6-4t))}{3(1-\phi)+4\gamma\phi t}.$$

Finally, if $\tilde{t}(\phi,\gamma)\le t\le\bar{t}(\phi,\gamma)$ then there exists a unique spot price equilibrium that solves

$$p_1(p_2)=\frac{(p_2+tv)(1-\phi)+2\gamma\phi tv}{2(1-\phi+2\gamma\phi t)};\quad p_2(p_1)=\tfrac{1}{2}(tv+p_1).$$

Solving the system of equations yields the spot period price equilibrium given in Lemma 5. When $\bar{t}(\phi,\gamma)<t\le 1$, there exist infinitely many spot period equilibria that solve the system of equations given in Lemma 5. Differentiating $\bar{t}(\phi,\gamma)$ with respect to ϕ and γ, we get that $\bar{t}(\phi,\gamma)$ is increasing in both. Taking the limit of $\bar{t}(\phi,\gamma)$ when $\gamma\to 0$ and using L'Hôpital's rule, we get

$$\lim_{\gamma_1\to 0}\bar{t}(\gamma_1)=\lim_{\gamma_1\to 0}\frac{d\bar{t}(\gamma_1)}{d\gamma_1}=\frac{2}{3}.\quad\square$$

Proof of Theorem 3. We focus on the parameter region $\tilde{t}(\phi,1)\le t\le 2/3$, where there exists a unique spot period equilibrium. The function $\Pi_1^T(\gamma)$ is continuous and differentiating it with respect to γ twice, we find that it is strictly concave for $t\le 1/2$ and strictly decreasing (and convex) for $1/2\le t\le 2/3$. Let $\hat{\gamma}$ be the solution to $d\Pi_1^T(\gamma)/d\gamma=0$. If $t\le 1/2$, the maximum γ^* may occur in $\gamma^*=\{0,\hat{\gamma},1\}$. The optimal γ is $\gamma^*=1$ if $\lim_{\gamma\to 1}d\Pi_1(\gamma)/d\gamma\ge 0$, which occurs if $t\le 2/7$ $\phi\le((-2+7t)/(6-17t+16t^2))$. The optimal γ is $\gamma^*=\hat{\gamma}$ if $\lim_{\gamma\to 1}d\Pi_1^T(\gamma)/d\gamma<0$ and $\lim_{\gamma\to 0}d\Pi_1^T(\gamma)/d\gamma>0$, which occurs if either $t\le 2/7$ and $\phi>\hat{\phi}$ or $2/7<t\le 1/2$ and $\phi>\hat{\phi}$, where

$$\hat{\phi}\le\frac{3(2-5t+4t^2)-2t\sqrt{3(19-44t+12t^2)}}{6-9t-8t^2}.$$

Otherwise, the maximum occurs at $\gamma^*=0$. $\square$

Endnotes

[1] Adding capacity constraints softens the competition between the firms and in the extreme the firms act as separate monopolists. Hence, we choose to ignore capacity constraints so as to emphasize the impact of competition.

[2] The results continue to hold qualitatively under more complicated value distributions, though the analysis becomes significantly more cumbersome.

[3] It is possible to derive these heterogeneous beliefs by assuming that customers receive a private signal on the firm they prefer. Assume that all customers share the same prior belief $(\frac{1}{2}, \frac{1}{2})$ for preferring each firm and let ω_i be the state of nature indicating that firm i is preferred, $i \in \{1,2\}$. At the start of the advance period, each customer observes a private signal on his preferred firm. In particular, consider the case where the signal $s = \{s_1, s_2\}$ is binary with accuracy α. That is, the conditional probability distribution is $\mathbb{P}\{s_i \mid \omega_i\} = \alpha$ and $\mathbb{P}\{s_i \mid \omega_j\} = 1 - \alpha$, if $i \neq j$ with $i, j \in \{1,2\}$. Customers are heterogeneous in the accuracy of their signal with $\alpha \sim U[\frac{1}{2}, (1+\delta)/2]$. The probability that a signal s_i is received by a customer with signal accuracy α is $\mathbb{P}\{s_i \mid \alpha\} = \frac{1}{2}\mathbb{P}\{s_i \mid \omega_1\} + \frac{1}{2}\mathbb{P}\{s_i \mid \omega_2\} = \frac{1}{2}$. Let $\gamma_i(\alpha)$ be the posterior belief that a customer with signal accuracy α prefers firm 1 after receiving signal s_i. Bayes' rule yields $\gamma_1(\alpha) = \alpha$ and $\gamma_2(\alpha) = 1 - \alpha$. Therefore, the posterior probabilities of a customer who observed signal s_i are γ_i for firm 1 and $1 - \gamma_i$ for firm 2. Upon observation of the realization s_i (which happens with probability $\frac{1}{2}$), a customer updates the belief from $\frac{1}{2}$ to γ_i, which results in the continuum of customer types described in the text.

[4] In the online appendix, we provide an analysis of a variant of the model, where consumers can purchase two units in advance, one from each firm. If consumers consider purchasing more than one unit, competition is dampened, which increases the benefit of advance selling. In particular, consumers purchase from both firms in advance if the equilibrium prices are low enough, which is the case when the degree of heterogeneity is low. In this case, advance selling to consumers at a low price may be desirable, because it eliminates the competition between firms—firms do not have an incentive to undercut each other's price if they can sell to the entire market. Nevertheless, as in the model we present in Section 4, spot selling is still preferred to advance selling in the majority of cases.

[5] The no profit result is closely related to the assumption that consumers only buy once. While allowing consumers to buy on the spot if they realize that they favor the other firm, will not result in zero profit, it still eliminates the benefit of advance selling. In this case, firms undercut each other's advance price until they are indifferent between selling in advance or only on the spot. Thus, the revenue obtained by advance selling is not higher than under spot selling. Allowing consumers to buy two units in advance eliminates the competition in the advance period when consumers are homogeneous, so that advance selling is preferable to spot selling. As we show in the online appendix, however, even when customers are allowed to buy from both firms in advance, advance selling is still inferior to spot selling in most cases with $\delta > 0$.

[6] While in equilibrium switchers have a higher average valuation than loyals, this does not limit the generalizability of the results. To explain, consider the extreme cases. If the average value of loyals is very high, a firm will ignore switchers in the spot period. The firm will effectively act like a monopolist and benefit from using advance selling. At the other extreme, if loyals have essentially zero values, a firm would ignore them and will not consider selling in advance. Therefore, the interesting dynamic, which we study, occurs in the intermediate situation in which both loyals and switchers are relevant to the firm in the sense that they have similar enough valuations so that both segments influence the degree of competition and equilibrium prices.

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