Pricing Capacity and Recourse Strategies: Facilitate Reselling, Offer Refunds, or Overbook?

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Abstract

Perishable capacity is often sold before it is used (e.g., tickets sold weeks before a sporting event) which creates the opportunity to include in the pricing mechanism a recourse strategy, i.e., allowing the firm or buyer to change ownership after an initial transaction. For example, a buyer could be allowed to resell the purchased unit to another buyer (e.g., a ticket exchange), or the firm could offer to refund the buyer if the buyer prefers to relinquish the unit (which is equivalent to selling an option to the buyer), or the firm could overbook, i.e., sell its capacity twice, possibly denying service to the first buyer (e.g., common practice among airlines). Recourse mechanisms tend to be controversial, both in terms of whether sellers should support them and how they impact buyer welfare. We find that recourse strategies are able to substantially increase the firm's profit and can at the same time increase buyer welfare. In fact, selling in advance may be a sub-optimal strategy without some form of recourse, especially in situations with ample demand relative to capacity. Among all selling mechanisms, reselling is optimal, but overbooking can be nearly as effective. Reselling is preferred even though consumers are able to sell for more than they paid. Consumer reselling also eliminates opportunities for speculators. We conclude that a firm selling capacity in advance should generally adopt some recourse strategy.

1 Introduction

Many firms sell perishable capacity to consumers, capacity that bundles a service with a particular moment in time. Examples include airline flights, hotel rooms, cruise ships, sporting events, music concerts, theatrical events and many others. In these markets consumers learn their preferences over time but are aware of their potential interest in the product well in advance of the moment of delivery: e.g., a person may know in January that she has an interest to take a cruise the second week of July. If the firm sells its capacity in advance, then what options are available to the firm or to consumers if they "change their mind", i.e., what recourse does the firm and/or consumers have if after making the initial commitment (e.g., a cruise ship booking), they want to modify their agreement before the delivery date? We define several options and identify the firm's optimal selling mechanism.

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In these settings the firm has roughly two approaches for timing of when to ideally sell to consumers: (1) sell on the "spot", close to the time of delivery, when consumers have resolved uncertainty over their preferences, or (2) sell in "advance", far ahead of the delivery time, when consumer preferences are somewhat uncertain. It has been shown that advance selling can be remarkably effective, despite the fact that the firm needs to offer a price discount to compensate the buyer for their preference uncertainty (Gale & Holmes 1993). It works because the firm can prefer a sure sale for less in advance over an uncertain sale for more later on.

If advance sales are allowed, then the firm needs to consider what happens if circumstances change (for the consumer or the firm) that may warrant a modification to the initial terms, i.e., a recourse strategy. Rather than leaving these situations to some ad-hoc renegotiation process, the firm and consumers can plan for these contingencies by explicitly including recourse options into the initial agreement, thereby allowing both parties to anticipate correctly the possible outcomes after the initial agreement. We consider all possible recourse strategies related to who owns the good: reselling, refunds and overbooking.[1] With reselling the firm allows the initial buyer to sell the unit to another consumer. With refunds the initial buyer can return the unit to the firm, who then can attempt to sell the unit to another consumer. (Refunds are equivalent to offering consumers an option with a non-refundable fee to purchase the option, and a price to take ownership of the unit that can be exercised after the value for the good is observed.) With overbooking the firm attempts to find a second buyer who is willing to pay more than the initial buyer. If that happens, then the firm takes back the unit from the initial buyer, compensates the buyer for this action (at a pre-specified fee) and then transfers ownership to the second buyer.

Given the breadth of possible price paths and recourse strategies, it is useful to define a framework to understand the firm's mechanism design options. At a high level, the firm's first goal is to generate value in the system. There are two means for generating value: (i) transfer ownership of the unit to some consumer because consumers receive some (though, maybe not much) value from the unit whereas the firm surely receives zero value from owning the unit at the end of the horizon; and (ii) conditional that a transfer is made, transfer the unit to the consumer who values it the most. Presuming some value is generated, the second goal of the firm is to extract some of that value for itself, i.e., to earn revenue. Unfortunately, there is tension among these goals, i.e., a selling mechanism cannot simultaneously maximize all forms of value generation and value extraction. For example, Myerson (1981) demonstrates that in a one period model the seller's optimal mechanism deliberately does not maximize the probability of a transaction (i.e., limits value

[1]There are other recourse mechanisms that do not involve ownership. For example, with a price matching guarantee the firm agrees to change the price paid after some information is learned (Lai *et al.* 2010, Huang *et al.* 2017). In our model there is no need/justification for such price adjustments. Alternatively, there could be a change in the quality of service offered to the customer (Biyalogorsky *et al.* 2005), such as a room or seat upgrade, but we do not include multiple types of products in our model.

creations) so as to increase the fraction of value the firm can extract - the firm is willing to risk not making a transaction because this increases the share of value the firm can earn if there is a transaction. This issue also arises when capacity is sold over time. To increase the probability of a transaction it is tempting to sell to the first willing buyer, but this may reduce the chance the buyer with the highest value actually uses the product.

The two price strategies, spot selling and advance selling, take different approaches to value creation and value extraction. Advance selling emphasizes the probability of some transaction and the fraction of value extracted at the expense of possibly not allocating the unit to the customer with the highest value. Spot selling increases the probability that the customers with the highest value receives the good, but accepts the risk that a transaction might not occur, thereby leaving the system with no generated value.

Recourse strategies are effective because they can increase the probability a transaction is made (relative to spot selling) and the probability the highest value consumer uses the product (relative to advance selling without recourse). But they do so in different ways. With reselling the initial buyer sells the unit only if doing so makes the buyer better off. Hence, a second transaction, if it happens, guarantees an increase in generated value. Refunds allow the initial buyer to return the unit if the buyer's value turns out to be low, thereby giving the firm the opportunity to possibly sell the unit to a buyer with a higher value. Overbooking allows the firm to try to sell to a buyer with a higher value than the initial buyer. On average this works, but it isn't guaranteed - the initial buyer may indeed have the highest value unbeknown to the firm. However, even if that were to occur, the buyer knows that the firm provides some compensation, thereby making the buyer willing to agree to the arrangement.

Among recourse mechanisms, reselling may be the most controversial. From the firm's perspective, it isn't clear why the firm would want to relinquish control over pricing to consumers. Doing so could allow speculators to enter the market and possibly lead to competition with consumers for some of the firm's inventory in the spot period, either of which may be detrimental to consumer welfare. Nevertheless, online marketplace companies such as Stubhub (owned by eBay), Ticketmaster (owned by Live Nation), RazorGator and many others have made the exchange of tickets through reselling safer and more efficient. The result has been a rapidly growing market which is expected to increase in value to about $15 billion by 2020 (Technavio 2015). As if the mere existence of these markets isn't enough evidence that they provide value, Michael Lewis (2019), in an empirical study of season ticket sales for a sports league, finds that reselling increases seller revenue and consumer welfare.

To preview our results, we find that advance selling without a recourse strategy is not always the firm's best choice. When demand is ample relative to capacity, it is more important to identify the buyer with the highest value than to ensure a transaction is made (which is likely given high demand). Hence, spot

selling can be preferred over advance selling. However, adding a recourse strategy to advance selling always makes selling in advanced the preferred mechanism. Among recourse strategies, reselling is best for the firm, followed by overbooking and then refunds. In fact, in our model reselling is the optimal mechanism among all possible mechanisms. For consumers, adding recourse mechanisms to advance selling improves their welfare. Finally, if a firm wants to prevent speculators, allowing consumers to resale is an effective deterrent - an efficient consumer reselling market eliminates any profit opportunity for speculators.

2 Related Literature

There is a large literature focused on selling capacity over time. Our work is distinctive because we integrate into a single model many mechanisms that have previously been treated separately. For example, there are models on various recourse mechanisms, but none that compare across mechanisms. Thus, they do not present a theory for the reasons why one mechanism is better than another and under what conditions. Furthermore, we identify reselling as an optimal mechanism without relying on exogenously (i.e., assumed) market frictions (e.g., constraints on pricing flexibility).

Especially when selling perishable capacity well in advance of its usage, it is possible that consumers are initially somewhat uncertain regarding their preferences. An advance selling strategy, despite this uncertainty, can be highly effective for the firm: Gale & Holmes (1993) show that advance selling allows a monopolist firm to price discriminate between consumers who are relatively indifferent across products (e.g., peak and off-peak flights) and those that have stronger preferences; Dana (1998) shows that advance purchase discounts can arise in a competitive market; DeGraba (1995) demonstrates that a firm can be better off selling a limited amount of capacity in advance to consumers unsure of their preferences; Xie & Shugan (2001) emphasize that advance selling can be effective even with ample capacity; Chu & Zhang (2011) find that it is always in the firm's interest to sell to consumers with less than perfect preference information; advance selling can be used to update a seller's demand forecast (Moe & Fader 2002; Chu & Zhang 2011; Li & Zhang 2013); and Cachon & Feldman (2011) show that advance selling via subscriptions can be effective even in services prone to congestion, despite the limited ability of subscriptions to control congestion. Nevertheless, some limitations of advance selling have been identified: Xie & Shugan (2001) and Prasad *et al.* (2011) show that advance selling is not optimal if marginal costs are high and Cachon & Feldman (2017) show that advance selling can harm firms by increasing the competitiveness of the market. We highlight another limitation. Advance selling is less effective for the firm than spot selling when demand is ample (relative to capacity). None of the mentioned studies on advance selling consider recourse mechanisms.

Among recourse mechanisms, reselling has drawn the most attention. Early work focuses on reselling by

individuals who do not value the firm's good, i.e., speculators. These resellers have been generally viewed as undesirable for a firm: Roth (2007) describes reselling by speculators as a repugnant transaction. For example, when late arriving consumers have higher valuations than early consumers, a firm might want to sell with an increasing price path. But Courty (2003) argues that speculators prevent the firm from implementing that strategy because they create competition to sell to the high value consumers. However, he does not consider the possibility of consumer reselling, possibly because at that time technology was not available to support an efficient consumer reselling market. In fact, in his model, introducing consumer reselling allows the firm to increase its revenue and eliminates speculators. We establish this result in our model as well.

Recent work suggests that a firm can benefit from speculators when the firm has restricted control over its pricing. In Su (2010) speculators indirectly allow a firm to implement dynamic pricing, and so exist only when the firm is unable or unwilling to adjust its prices. In our model there are no restrictions on what prices can be charged, so speculators play no role (i.e., they are unable to enter and earn a profit). In Cui *et al.* (2014) speculators serve as a low-cost vehicle to transfer units from consumers with low value to consumers with high value. Our results do not rely on the existence of transaction costs. In general, it is not clear why speculators should play a major role in an efficient market. Like the firm, speculators have zero value for the good, and therefore face a disadvantage in the resale market relative to a seller that does value the good, i.e., speculators are not as willing to pay as much for the good as consumers who have access to reselling. And speculators are likely to have inferior market data relative to the firm for setting appropriate prices. Hence, speculators are more likely to exist in markets with significant trading frictions. The availability of inexpensive information technology has likely reduced these frictions, thereby enabling efficient consumer-to-consumer reselling exchanges (e.g., StubHub).

As in our model in which consumers arrive sequentially, Yang *et al.* (2017) consider reselling positions in a queue. However, consumers in their model do not learn information over time regarding their valuation and they do not consider dynamic pricing. Nevertheless, in their setting they demonstrate that social welfare and firm profits can increase substantially by allowing consumers to resell.

Some work considers refunds as a recourse mechanism. As in our model, Xie & Gerstner (2007) and Gallego & Sahin (2010) study a monopolist selling to consumers who begin the selling horizon somewhat uncertain of their preferences. With a refund a consumer pays the full price upfront but can receive a partial refund if the consumer later learns of a low value for the good. Equivalently, this can be implemented using options - the consumer pays a non-refundable fee for the option to purchase, and an exercise fee later on if the consumer decides to purchase. Both papers show that refunds/options can increase the seller's revenue but neither considers alternative recourse mechanisms. Cui *et al.* (2014) recommends offering options to prevent

speculators from entering the market. Guo (2009) extends Xie & Gerstner (2007) to a competitive setting and demonstrates that refunds may no longer be offered, thereby suggesting that competition is a reason for the limited use of refunds in practice. We offer an alternative explanation for the narrow application - refunds are the least effective of the recourse mechanisms for the firm.

Overbooking is the practice of selling beyond capacity: e.g., selling more tickets than seats on a flight, or more reservations for a hotel than rooms, or scheduling more appointments in a day than a doctor could actually deliver. Most research focuses on overbooking as a strategy to mitigate the consequences of customers who do not "show up" to actually use the good they purchased: e.g., Weatherford & Bodily (1992), Biyalogorsky *et al.* (1999), Karaesmen & Van Ryzin (2004). In our model overbooking is used by the firm as a tool to find the highest paying customer. Gallego *et al.* (2008) refer to this form of overbooking as a "callable product" and Biyalogorsky *et al.* (1999) call it "overselling". Both demonstrate that it can increase a firm's revenue. The same is true in our model, but we also demonstrate that overbooking is not as effective for the firm as reselling.

Recourse strategies are valuable only if some information changes over time. In our model there are two sources of evolving information. The critical one is that consumers learn information about their preferences over time. For example, after making the initial purchase, a consumer might discover their value for the good is low, or that some other consumer would be willing to pay even more. The second source of information is the amount of demand - for any given posted price their is demand uncertainty. There is a large number of studies, like spot selling, that avoid (or ignore) the first source of uncertainty regarding preferences and focus only on the second (the amount of demand): Aviv & Pazgal (2008), Liu & van Ryzin (2008), Cachon & Swinney (2009). They find that consumers may be willing to pay a premium early on in the selling season to avoid the risk of being unable to make a purchase later on because no more inventory remains. Rationing risk can be present in our model, and we show that rationing risk works in the firm's favor - the initial buyer is willing to pay more to prevent being excluded from the good. However, we also demonstrate that our results do not depend on the presence of rationing risk - the initial buyer can be willing to purchase early on, despite uncertain preferences and no rationing risk, because the firm offers a discount, and the firm offers a discount because transferring ownership to the initial buyer is valuable.

3 Model Description

We study a model in which a firm sells perishable capacity to consumers, such as admission to an entertainment event, transportation services or some form of lodging. Demand is uncertain and capacity is potentially restrictive (i.e., demand may exceed supply). The capacity is used at a particular point in time, and con-

sumers can anticipate ahead of that time their need for the capacity, albeit with clearer preferences closer to the time of the event than well in advance. Consequently, the firm can sell its capacity over time, e.g., well in advance or closer to "on the spot" (i.e., just before when the capacity is used). As a result, many selling mechanisms are feasible. The remainder of this section details the specifics of the model.

A single firm sells one unit of capacity that can be used by a single consumer at the end of a two period horizon. Period 1 is referred to as the advance period and period 2 is the spot period. The firm incurs zero marginal cost to deliver the unit. If the unit is not purchased by a consumer over the two periods, then the capacity is wasted, i.e., the firm receives no value for unsold capacity.

A single buyer, called buyer A, arrives in period 1 and remains to period 2. Buyer A's value for the good, V_A, is distributed on the interval $[0,1]$ with distribution and density functions $F(v)$ and $f(v)$. Let $\mu_A = E[V_A]$. Let v_A be the realization of V_A. Buyer A observes v_A only at the start of period 2, i.e., in period 1 buyer A only knows the distribution function for V_A, $F(v)$. For example, the buyer may want to celebrate a daughter's birthday at a basketball game, is unsure (in advance) if she will be able to attend, but knows that this uncertainty is resolved later (in the spot period). (See Papanastasiou & Savva (2017) and Feldman *et al.* (2019) for models in which consumer learning is endogenously determined by the firm's actions rather than, as in our model, an exogenous process.)

In period 2 there is a set of potential buyers, call them the B buyers, and all know their value for the good. To model demand in period 2, let V be the random variable representing the maximum value among the B buyers. Let v be the realization of V. Let $G(v)$ and $g(v)$ be the distribution and density functions of V. Let $G^+(v)$ be the distribution function for the maximum value across the entire set of buyers,

$$G^+(v) = F(v)\,G(v).$$

We assume that $F(v)$ and $G(v)$ have increasing generalized hazard rates, i.e., $vg(v)/(1-G(v))$ is strictly increasing.

Buyer A always has the option to wait to period 2 to attempt a purchase. The advantage of doing so is that buyer A observes v_A in period 2. The disadvantage is that buyer A may risk that the good is sold to a B buyer. In particular, if the buyer waits to period 2 to attempt to purchase, then buyer A anticipates some subset of the B buyers have the opportunity to purchase before buyer A thereby creating some rationing risk for buyer A. However, if none of the B buyers can purchase ahead of buyer A, then buyer A faces no rationing risk. Let $\hat{V}$ be the random variable for the maximum value among those B buyers that are ahead. Variables and functions associated with $\hat{V}$ are identified with a carat: e.g., $\hat{G}(v)$ is the distribution function of $\hat{V}$. As with $G(v)$, assume $\hat{G}(v)$ has an increasing generalized hazard rate. Naturally, the following stochastic

ordering holds for all v, i.e., $G(v) \leq \hat{G}(v)$.

The parameters and sequence of events are common knowledge to the buyers and the firm. All agents are risk-neutral, utility maximizers and correctly anticipate future actions. The firm's objective is to design the terms of trade to maximize expected revenue (which is equivalent to expected profit given the zero marginal cost for delivering capacity).

4 Selling Mechanisms

The interesting strategic interaction in this model occurs between the firm and buyer A. Several selling mechanisms are considered. With each of these mechanisms, the seller attemps to sell via a posted price and that in case there is more than one interested buyer, the allocation is random (all results qualitatively follow through with other allocation rules, e.g., efficient allocation.) With the simplest mechanism, called "spot selling", the firm sells only in period 2 to the full set of consumers (A and the Bs). With "advance selling", the firm attempts to offer buyer A a price in period 1 that induces buyer A to purchase despite having uncertain preferences. The next three mechanisms combine advance selling with some form of recourse, which specifies what can be done after an initial transaction agreement.

With "reselling", if buyer A purchases in advance, then the buyer is authorized to attempt to resell the unit to a B buyer in period 2 via a posted price. (Results apply to any period 2 selling mechanism, so we assume the intuitive posted price mechanism.) The firm can specify upfront a transfer fee t_r that buyer A must make to the firm if the buyer is able to sell the unit.

With "refunds", buyer A is given the option to return the unit to the firm at the start of period 2 after observing v_A. This gives buyer A some downside protection - if the buyer observes a very low v_A, then the buyer can at least request a refund t_c ("c" for cancellation). If the unit is returned, the firm has the opportunity to try to sell it to one of the B buyers in period 2.

The third option, "overbooking", allows the firm to sell its capacity twice - if the firm is able to sell the unit to a B buyer in period 2, even though it was sold to buyer A in period 1 the firm has the right to take back the unit from buyer A (and sell it to the B buyer) for a prespecified amount of compensation, t_o. In our model overbooking is not done to hedge the risk of a buyer not "showing up" to use the capacity the buyer purchased - buyer A uses the capacity unless explicitly denied service.

The set of considered recourse strategies spans the feasible options - with reselling buyer A owns and sets the period 2 offered price, with refunds the firm owns and sets the period 2 offered price, and with overbooking buyer A owns the unit (in the sense that it retains posession if there is no period 2 sale) but the firm sets the period 2 offered price.

With each mechanism we evaluate the firm's optimal contract offer, the firm's optimal revenue, and the expected total surplus (i.e., social welfare), which is the expected value generated by the capacity. Surplus depends on two factors: (i) the probability the unit is transferred to a buyer (no value is generated unless a transfer is made) and (ii) the probability it is consumed by the buyer with the highest value (more value is generated if the consumer with the higher value gets the unit). Prices affect surplus only indirectly by changing those probabilities. Surplus is of interest for government policy, and therefore is of interest to the firm.

4.1 Spot selling

With a sufficient high period 1 price, $p_1 = 1$, the firm can ensure that it only sells in period 2 to the full set of buyers. Let $\Pi_s(p_2)$ be the firm's revenue with spot selling with a posted price p_2,

$$\Pi_s(p_2) = \left(1 - G^+(p_2)\right) p_2$$

Let p_s^* be the firm's optimal spot selling price:

$$p_s^* = \arg\max_{p_2} \Pi_s(p_2)$$

There is a unique price to maximize buyer A's profit because, from Corollary 2 in Lariviere (2006), $G^+(p_2)$ has an increasing generalized failure rate.

If at the end of period 1 buyer A does not own the unit, then buyer A can anticipate the opportunity to purchase it in period 2. Buyer A's expected utility at that time is

$$W_2 = \hat{G}(p_s^*) E\left[(V_A - p_s^*)^+\right] :$$

the B buyers ahead of buyer A in the purchase queue must not purchase, with probability $\hat{G}(p_s^*)$, and then buyer A earns expected value $E\left[(V_A - p_s^*)^+\right]$. If $\hat{G}(p_s^*) = 1$ then buyer A faces no rationing risk.

For a distribution function, $H(x)$, define

$$\delta_H(v) = \frac{1 - H(x)}{1 - F(x)} E\left[(V_A - x)^+\right].$$

It follows that

$$E\left[(V_A - p_s^*)^+\right] = \delta_F(p_s^*).$$

The firm earns surplus $\Pi_s(p_s^*)$ and consumer surplus is $\delta_{G^+}(p_s^*)$. Total surplus with spot selling is $S_s = \Pi_s(p_s^*) + \delta_{G^+}(p_s^*)$

4.2 Advance selling without recourse

The firm can attempt to sell the unit to buyer A in period 1 for price p_1. If buyer A purchases the unit, then buyer A owns it, otherwise, the firm can attempt to sell the unit in period 2 to the full set of buyers.

Buyer A can purchase in period 1 and receive expected utility $\mu_A - p_1$ or wait to attempt to purchase in period 2 and earn utility W_2. The maximum price buyer A pays in period 1 to purchase is $p_1 = \mu_A - W_2$. With that price, let Π_a be the firm's earning :

$$\Pi_a = p_1 = \mu_A - W_2$$

Total surplus is $S_a = \mu_A$ because the unit is guaranteed to transfer to buyer A, who has an expected value of μ_A. The firm's surplus is Π_a, and buyer A's surplus is W_2.

4.3 Reselling

In the reselling mechanism the firm has a single unit to sell and reselling is allowed. Let $u_2(p_2, v, t)$ be the expected utility of a seller in period 2 that offers price p_2, has value v for the good, and pays transfer fee t_r,

$$u_2(p_2, v, t_r) = (1 - G(p_2))(p_2 - t_r) + G(p_2)\, v.$$

To explain, if at least one of the B buyers values the unit at p_2 or greater, a transfer is made and buyer A obtains $p_2 - t_r$. Otherwise, the unit remains with buyer A. Let $p_2(v, t_r)$ be the unique price that maximizes $u_2(p_2, v, t_r)$. (It is unique because G has an increasing generalized hazard rate.) For buyer A, the expected value of owning the unit in period 1 is $U_r(t_r)$,

$$\begin{aligned} U_r(t_r) &= E_V\left[u\left(p_2(v, t_r), v, t_r\right)\right] \\ &= \int_0^{1-t_r} u_2\left(p_2(v, t_r), v, t_r\right) f(v)dv + \int_{1-t_r}^{1} v f(v)dv. \end{aligned}$$

Buyer A does not know v_A in period 1, but does know that there may be something to gain from reselling in period 2.

The firm can induce buyer A to purchase in period 1 with the following price

$$p_r(t_r) = U_r(t_r) - W_2$$

The firm's earning are $\Pi_r(t_r)$,

$$\Pi_r(t_r) = p_r(t_r) + T(t_r) = U_r(t_r) + T(t_r) - W_2$$

where $T(t_r)$ is the expected transfer fee buyer A pays in the resale market

$$T(t_r) = t_r \int_0^{1-t_r} (1 - G(p_2(v, t_r))) f(v) dv$$

The firm's surplus is $\Pi_r(t_r)$, buyer A's surplus is W_2 and the surplus for the B buyers is $S_r(t_r)$

$$S_r(t_r) = \int_0^{1-t_r} \delta_G(p_2(v, t_r)) f(v) dv$$

Although it might be tempting for the firm to try to profit from buyer A's reselling in period 2, according to Lemma 1, the firm should allow buyer A to earn as much as possible in the resale market because then the firm can extract that value from the buyer via the period 1 price.

Lemma 1. *With a reselling contract, the firm's optimal transaction fee is* $t_r = 0$.

There has been significant debate on the practice of reselling. A major concern is the presence of speculators who purchase capacity in advance with the hope of later selling the capacity at a higher price. Such behavior is viewed at best as a loss of revenue to the firm, and at worst as a violation of ethical norms. And indeed, it is not immediately clear that speculators cannot enter the market. Unlike buyer A, who is limited to sell to only the B buyers in the resale market, the speculator can sell to all buyers, including buyer A: the speculator can generate more revenue in the resale market than buyer A. Furthermore, the firm cannot charge buyer A the full amount buyer A expects to earn from purchasing the unit, $U_r(0)$, because buyer A has the option to wait to period 2 to purchase the unit and earn W_2. However, according to Lemma 2 those advantages for the speculator are outweighed by the disadvantage: buyer A values the good (and is therefore willing to pay a portion of that value) whereas the speculator has no value for the good; and the speculator does not have an informational advantage over the other agents in the market.

Lemma 2. *Speculators are not able to enter the reselling market.*

Lemma 2 emphasizes that speculators can only enter a market that faces clear inefficiencies, such as rigid prices. Therefore, the presence of speculators should be viewed as a symptom of a market failure because they provide no intrinsic value to the system - they neither create the capacity nor the value from its use. In our model, due to the absence of market frictions, there is no value for speculators and they are unable

to profitably enter. Instead, reselling is done by the consumers themselves, as is currently observed in many markets. Unlike the firm, the consumer knows the value for the capacity and therefore is better able to set a resale price. Nevertheless, the firm can anticipate this value and price accordingly with the initial transaction. In effect, the firm is able to extract from buyer A the entire additional value the consumer could earn from product resale, thereby benefiting from resale as well.

4.4 Refunds

With a refund mechanism the firm offers a refund t_c to buyer A if the buyer wants to return, or "cancel", the unit at the start of period 2 after the buyer observes v_A. Clearly, buyer A returns the unit if $v_A \leq t_c$ and otherwise keeps it. This mechanism is equivalent to an option: buyer A pays in period 1 a non-refundable amount $p_1 - t_c$ for the option to purchase the unit in period 2, after observing v_A, for the exercise fee t_c. These outcomes are equivalent to the refund mechanism - if the option is exercised, then the buyer's total cost is p_1, but if not exercised, then the buyer incurs a net loss of only $p_1 - t_c$. For simplicity, we assume that this mechanism is presented in the form of a refund rather than an option.

If buyer A returns the unit to the firm, then the firm sells the unit in period 2 to the B buyers. Let $\pi_2(p_2)$ be the firm's profit in period 2 selling to the B buyers:

$$\pi_2(p_2) = (1 - G(p_2))\, p_2$$

Let p_2^* be the firm's (unique) optimal price:

$$p_2^* = \arg\max \pi_2(p_2)$$

and $\pi_2^* = \pi_2(p_2^*)$ the firm's optimal profit. Note, p_2^* is the optimal period 2 price when selling only to the B buyers, and p_s^* is the optimal period 2 price when selling to the full set of buyers (i.e., buyer A included).

Buyer A's utility from owning a refund contract is

$$U_c = \int_0^{t_c} t_c f(v)\, dv + \int_{t_c}^{1} v f(v)\, dv = \delta_F(t_c) + t_c :$$

If the buyer's value is lower than the refund t_c, the buyer returns the unit, and otherwise keeps it. Buyer A pays at most p_c for the refund contract:

$$p_c = U_c - W_2$$

The firm's profit from selling a refund contract is

$$\Pi_c(t_c) = p_c + F(t_c)(-t_c + \pi_2^*)$$

Theorem 1. *With a refund contract the unique optimal refund is* $t_c^* = \pi_2^*$.

According to Theorem 1 the firm's optimal refund, t_c^*, equals the firm's expected period 2 earning with the unit. Hence, if buyer A returns the unit to the firm, the firm at that point earns nothing from the transaction. Put another way, the firm gives buyer A the best possible deal buyer A could receive from returning the unit. Hence, the firm makes up for breaking even on the return transaction through the upfront price, p_1.

The firm's optimal profit with the refund contract is $\Pi_c(t_c^*)$. Total surplus is

$$S_c = \Pi_c(t_c^*) + W_2 + F(t_c^*)\,\delta_G(p_2^*)$$

4.5 Overbooking

With an overbooking mechanism the firm sells to buyer A and then tries to sell the unit again to one of the B buyers. If a B buyer purchases the unit, the firm denies service to buyer A but refunds the buyer the prespecified amount, t_o.

The firm's earnings in period 2 are

$$\pi_2(p_2, t_o) = (1 - G(p_2))(p_2 - t_o) :$$

the amount the firm needs to pay buyer A is analogous to a transaction cost self-imposed on the firm. Let $p_2(t_o)$ be the firm's (unique) period 2 price to offer to the B buyers.

In period 1 buyer A's utility from the contract is

$$U_o(t_o) = \mu_A G(p_2(t_o)) + (1 - G(p_2(t_o)))\,t_o,$$

where the first term is the utility if the unit remains with buyer A and the second term is buyer A's utility if the unit is transferred to another buyer. The buyer could always wait to period 2 to purchase and earn utility W_2. Hence, the most the firm can charge in period 1 is

$$p_1(t_o) = U_o(t_o) - W_2$$

The firm's revenue from the overbooking contract is

$$\Pi_o(t_o) = p_1(t_o) + \pi_2(p_2(t_o), t_o)$$

The optimal overbooking mechanism for the firm makes buyer A indifferent between consuming the unit and relinquishing it back to the firm, $t_o = \mu_A$ (Theorem 2). Hence, the firm does not assert its power through the transfer fee, but rather through the up front price, p_1, which is lower than the tranfer fee.

Theorem 2. *The firm's unique optimal overbooking mechanism fully compensates the buyer for the expected value of the good,* $t_o^* = \mu_A$. *Moreover,* $t_0^* > p_1$.

According to Theorem 2, the firm fully compensates buyer A for the expected value of the unit if the unit is taken from the buyer. Thus, the firm sets the period 2 price to maximize revenue given the information that it has, i.e., that the cost of making a sale to a B buyer is buyer A's expected value, μ_A. In period 1, the firm captures from buyer A this expected value, μ_A, minus buyer A's reservation utility, W_2. While buyer A is indifferent between keeping the unit and giving it up in expectation, if asked to reliquish the unit, the firm compensates the buyer for more than the buyer paid for it.

Total surplus is

$$S_o = \Pi_o(t_o^*) + W_2 + \delta_G(p_2(t_o^*))$$

4.6 Mechanism comparison

Our primary finding, according to Theorem 3, is that for the firm there is a strong preference ordering across four of the selling mechanisms.

Theorem 3. *The firm's strict preference ordering across the following mechanisms (from least to most) is spot selling, refunds, overbooking and reselling, i.e.,* $\Pi_s(p_s^*) < \Pi_c(t_c^*) < \Pi_o(t_o^*) < \Pi_r(0)$.

Refunds are superior to spot selling because a refund mechanism with $p_c = t_c = p_s^*$ replicates spot selling, and that is not the best refund mechanism.

Overbooking is preferred to refunds because with refunds the firm is required to take back the unit from buyer A if buyer A so chooses (because the buyer observes a low value). With overbooking, the firm has the opportunity (rather than the requirement) to take back the unit from buyer A (if it is able to sell it in period 2). Consequently, with overbooking the unit transfers only when system value increases, whereas a transfer may occur with refunds that reduces system value.

Reselling is preferred over overbooking because with overbooking the firm sells to the B buyers only knowing the buyer A's average value, μ_A, whereas with reselling buyer A sells to the B buyers precisely

knowing v_A. Hence, buyer A can generate more surplus in period 2 with reselling than the firm can with overbooking. The firm extracts that additional surplus via its period 1 price, leaving it better off with reselling.

Advance selling without recourse is absent from Theorem 3. Section 4.6 establishes that advance selling may or may not be better than even spot selling. However, given that $0 < t_c^*$, it follows immediately that all forms of recourse increase the firm's revenue, i.e., $\Pi_a < \Pi_c(t_c^*)$.

Theorem 3 does not identify the firm's optimal selling mechanism. It establishes the ranking of the mechanisms we explore, but it does not identify the best selling mechanism among all possible mechanisms. The next theorem determines that reselling is in fact the firm's optimal mechanism. Consequently, there is no need to combine recourse mechanisms: e.g., the pairing of reselling with refunds or overbooking cannot do better than reselling alone.

Theorem 4. *The reselling contract is the firm's optimal contract.*

The theorem doesn't identify the optimal period 2 selling mechanism (nor does it need to). Myerson (1981) demonstrates that an auction with a reserve price maximizes the seller's revenue. However, a posted-price mechanism (as we assume) may be better if there are substantial costs to implement an auction (e.g., customers must wait for the auction event).

The remainder of the section reports on numerical comparisons of the mechanisms for a broad range of parameters. Let buyer A's value distribution, $F(v)$, be a beta distribution with parameters α and β. Hence, $\mu_A = \alpha/(\alpha+\beta)$. When $\alpha = \beta$, the beta distribution is symmetric about 1/2. The uniform $[0,1]$ is a special case of the beta distribution with $\alpha = \beta = 1$.

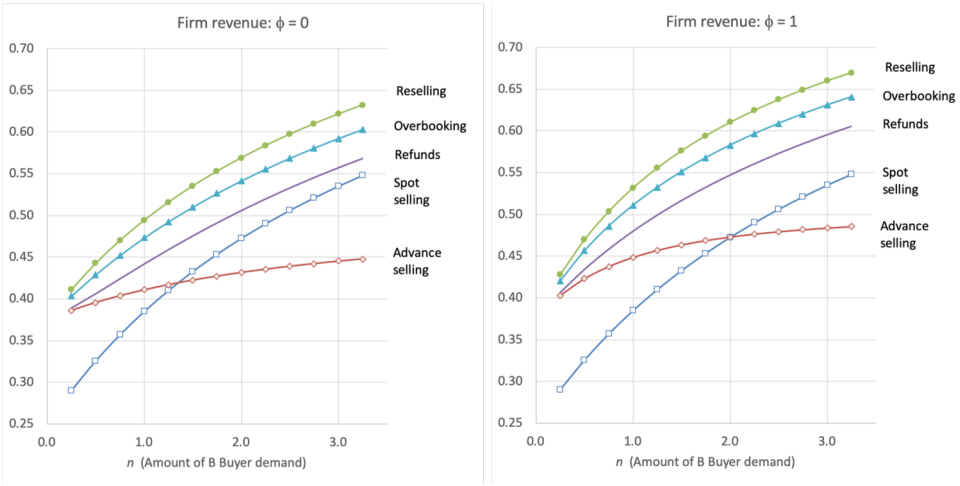
Let $G(v) = (F(v))^n$ for some parameter $n \geq 0$. Note, $G(v)$ is the distribution function of the maximum of n B buyers who have the same utility distribution as buyer A. Although it is most natural to interpret n when it is an integer, we allow it to be a continuous parameter. It follows that expected demand in period 2 is increasing in n.

To model the potential purchase queue in front of buyer A in period 2, let $\hat{G}(v) = (F(v))^{\phi n}$ for $\phi \in [0,1]$. When $\phi = 0$ then buyer A faces no rationing risk to wait to period 2 because the buyer is given the first opportunity to purchase in the period. Buyer A faces the maximum rationing risk when $\phi = 1$, because then buyer A has the least priority in the purchase queue.

Closed formed solutions do not exist for general values of α, β, and n. But numerical solutions are easily evaluated. The majority of this section presents results for $\alpha = \beta = 1$. More extreme cases are discussed at the end.

Figure 1 displays the firm's revenue across the considered mechanisms for different values of n and two

Figure 1. Firm revenue across different selling mechanisms with either no rationing risk (left panel, $\phi = 0$) or maximum rationing risk (right panel, $\phi = 1$) for buyer A, with $\alpha = \beta = 1$ (i.e., all buyer preferences are uniformly distributed on the interval $[0, 1]$)



levels of rationing risk when all buyers have uniformly distributed preferences, $\alpha = \beta = 1$.

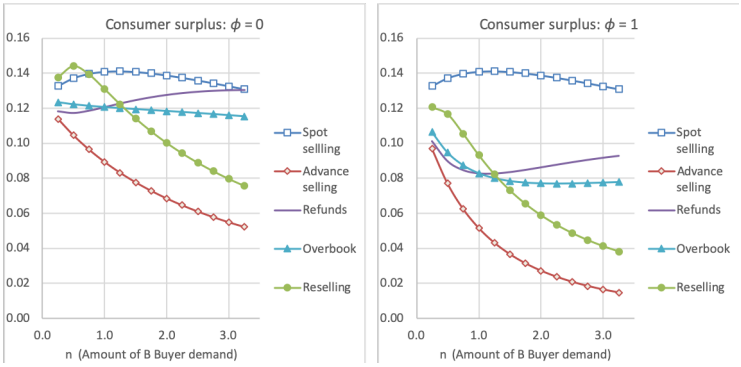
The level of demand relative to capacity influences which of the two non-recourse strategies is preferred - if demand is somewhat limited ($n < 1.33$), then advance selling is preferred, but if demand is ample ($n > 2$) then spot selling is preferred. With limited demand it is more important to ensure a transfer to some buyer (so the capacity is used) than to find (and to sell to) the buyer with the highest value. But when demand is ample, there is more value in finding the buyer with the highest value and less risk of unused capacity, so spot selling becomes more attractive. Xie & Shugan (2001) argue that advance selling is effective even when demand is limited (low n), but we find that advance selling (on its own) is only effective when demand is limited. Moreover, advance selling is preferred to spot selling over a wider range of demand, n, if the rationing risk grows (i.e., ϕ increases) because if buyer A's risk of waiting increases, the firm does not need to discount the period 1 price by much to entice the buyer to purchase in advance.

From the data in Figure 1, reselling yields a substantial 6.2% to 29% more revenue than the best non-recourse mechanism. Reselling also generates between 1.8% to 5.1% more revenue than overbooking.

Lemma 2 establishes that speculators cannot enter the market when the firm allows consumer reselling. But when demand is sufficiently ample ($n > 2.25$), speculators can enter the market when the firm uses advance selling without a recourse method. However, in those cases the firm prefers spot selling over advance selling without recourse. Hence, speculators are even unable to enter the market when the firm uses its best non-recourse mechanism.

Figure 2 displays consumer surplus with each of the mechanisms for the two extreme cases of $\phi = 0$ (no rationing risk) and $\phi = 1$ (maximum rationing risk). For consumers, advance selling without a recourse

Figure 2. Consumer surplus across different selling mechanisms with either no rationing risk (left panel, $\phi = 0$) or maximum rationing risk (right panel, $\phi = 1$) for buyer A and $\alpha = \beta = 1$.



option is by far the worst among the mechanisms - buyer A surely receives the unit, but pays a dear price because the buyer's only defense against the firm is the threat to wait to period 2. Relative to advance selling, all of the recourse mechanisms increase consumer surplus, in part because they give buyer A options, and in part because they allow for the possibility of a transfer to a B buyer with higher value. From the consumer perspective no one mechanism dominates over the entire parameter space. Most often, the best mechanism for consumers is spot selling because by waiting to the spot period the firm gives itself only one opportunity to sell the good which benefits consumers. But reselling may be the best for consumers (as well as for the firm) if the market of buyers B and buyer A's rationing risk are small. In this parameter space, buyer A's benefit from reselling is lower and the incentive to wait is higher, so the firm sets a relatively low advance price, which benefits buyer A.

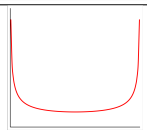
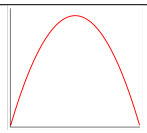
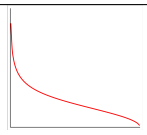
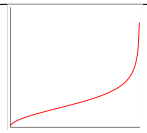
Table 1 indicates the other value distributions considered and the resulting percentage increase in revenue from the best non-recourse mechanism to reselling. The potential revenue improvement with reselling is greatest when the value distribution skews to the left ($\alpha = 3/4$, $\beta = 3/2$, $\mu_A = 1/3$, long upper tail), and least when the value distribution skews to the right ($\alpha = 3/2$, $\beta = 3/4$, $\mu_A = 2/3$, long lower tail). However, the results are incredibly robust as in all cases reselling is capable of a significant percentage increase in revenue for the firm.

5 Discussion

Our model is intentionally parsimonious to illustrate key distinguishing factors across these selling mechanisms. Nevertheless, several worthwhile extensions are worth consideration.

In our model buyer A's utility is independent of when buyer A purchases the unit. Hence, buyer A can be patient. In practice, some consumers value knowing in advance that they have access to a service. For

Table 1. Utility distribution functions evaluated and the percent revenue increase from reselling over the best non-recourse mechanism. In all cases there is no rationing risk, i.e., $\phi = 0$

α	β	μ_A	$f(v)$	Percent revenue increase
0.5	0.5	0.5		$[6.2, 29.0]$
2	2	0.5		$[5, 20.2]$
3/4	3/2	1/3		$[7.8, 30.1]$
3/2	3/4	2/3		$[4.7, 17.0]$

example, if buyer A is unable to purchase the unit in period 1 then the buyer may choose to not participate in the period 2 market, or may value period 2 participation less. In the content of our model, this preference would reduce the utility buyer A receives from waiting to purchase, W_2, which allows the firm to increase its period 1 price. Hence, this preference for early resolution, beyond the issue of rationing risk, makes advance selling more attractive (with or without recourse) relative to spot selling.

Two other features of buyer A's preferences are important. First, the firm knows the distribution of buyer A's preference precisely, which allows the firm to set a period 1 price equal to buyer A's expected utility, μ_A, minus buyer A's reservation utility, W_2. If buyer A's utility is actually slightly lower than what the firm expects, say $\mu_A - \epsilon$ for a small ϵ, then firm may select an advanced price in period 1 that is slightly too high. If buyer A doesn't buy in advance, then the advantages of advance selling are lost. The firm can protect itself from this risk by lowering its period 1 price, but doing so makes advance selling less attractive.

Second, it is presumed that the firm knows when buyer A learns its true utility for the good. In particular, the firm knows that buyer A has not observed v_A in period 1 and observes v_A in period 2. But what if such precise knowledge isn't possible? For example, say in period 1 there is β probability that buyer A observes v_A. Our analysis assumes $\beta = 0$. But if buyer A knows v_A and yet the firm chooses a price p_1 that presumes buyer A hasn't observed v_A, then the firm is choosing a price that is too low - the firm is offering a discount to compensate for buyer A's preference uncertainty that may not exist. See Authors (2020) for a full analysis of that situation.

Among recourse mechanisms, reselling is the best. In fact, according to Theorem 4, reselling is the

firm's best mechanism. Consequently, there is no need to combine recourse mechanisms: e.g., the pairing of reselling with refunds or overbooking cannot do better than reselling alone. But if reselling is optimal, why don't firms always allow resale in practice? We offer several explanations.

Timing: In our model buyer A observes v_A sufficiently in advance of when the unit is consumed to allow for a market to occur in which it is possible to transfer the unit to some B buyer (i.e., period 2). If buyers tend to observe their value sufficiently late (i.e., close to the time of consumption), there may not be enough time to effectively locate another buyer, which would reduce the value of selling in advance and surely the value of conducting a resale market. For example, if the good is a ticket on a plane and buyer A tends to observe v_A only a few days before the flight, then there may not be enough time to conduct a period 2 resale market. Furthermore, if as suggested above, consumers have a preference for knowing in advance that they have access to the good, then there may be a limited number of consumers willing to participate in the period 2 resale market. In these situations, overbooking may be preferred for two reasons. First, and foremost, with overbooking the firm does not have to wait until buyer A observes v_A to begin selling to the B buyers in period 2. This allows the firm to extend the duration of the period 2 market, a potentially significant advantage over reselling. Second, if recourse does need to occur quickly, then the firm is likely to be better able than buyer A to quickly find a B buyer with the firm retains control over the transfer process with overbooking.

Transaction costs: We have not included actual transaction costs. (A transfer between buyers and the firm does not reduce total surplus, but actual transaction costs do.) In practice, reselling requires consumer interaction and an ability for sellers to find potential buyers. To facilitate these matchings, firms can operate resale websites (this is commonly done by sport teams–all four major leagues now have sponsored resale marketplaces) or allow the use of third-party platforms (e.g., Stubhub and Ticketmaster), all of which involve various actual costs. Refunds require some communication between consumers and the firm. Overbooking does not involve matching costs, and the communication costs are minimal, but it does impose non-trivial psychological costs - even though buyer A might anticipate being denied service, in practice it is possible that this event imposes additional disutility beyond what is included in the model. Naturally, any of these transaction costs introduce market inefficiencies that reduce the value of a recourse mechanism. In addition, the final ranking of these mechanisms could change depending on the differences in transaction costs. For example, if actual transaction costs with overbooking are substantially lower than with reselling, overbooking could be preferred. And if transaction costs across all recourse mechanisms are too high, the firm might prefer a mechanism without recourse.

Consumer capability: Our buyer A is fully capable to choose a price in the resale market to maximize the buyer's utility. In practice, buyer A may lack the necessary information (e.g., the correct distribution of

what the B buyers are willing to pay) or the needed skill to evaluate the best price. Furthermore, buyer A may not be able to fully anticipate the value of reselling. For example, the endowment effect (Kahneman *et al.* 1990) predicts that once ownership is acquired the buyer may value having it more. The buyer is unlikely to anticipate this endowment effect, thereby denying the firm access to some value. In general, if buyer A does not have access to the same selling mechanism in period 2 as the firm (due to capabilities or other constraints), then the firm may not want buyer A to be responsible for reselling.

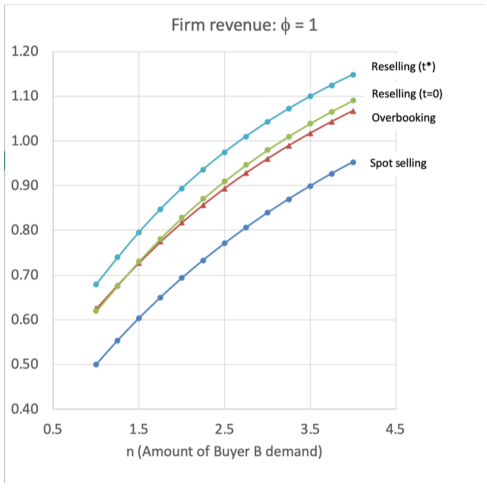
Another possible concern with reselling is competition in the period 2 market. With only one unit in the market, the seller in period 2 never faces competition. With multiple units owned by multiple agents, competition in the resale market might reduce the firm's revenue both from the units it tries to sell as well as through the firm's ability to capture some of the value buyer A gains from the resale market. Overbooking avoids competition because the firm retains full control over pricing and resale by consumers is prevented. To explore the possible impact of competition on reselling, we extend our model so that the firm begins with two units. If one unit is sold in advance to buyer A in period 1, then the firm and buyer A would compete in the resale market in period 2 (assuming the firm allows it). Details for each mechanism in this situation are provided in Appendix B.

When the firm does not face competition in the resale market, the firm prefers to maximize buyer A's earnings in the resale market. Hence, it is counterproductive to impose a transfer fee on buyer A. That logic is no longer valid when the firm and buyer A compete in the resale market. The firm remains interested to increase buyer A's earnings, but the firm does not want buyer A to interfere with the firm's effort to sell off its remaining inventory. The former argues for a low transfer fee, but the latter suggests that a positive transfer fee can be useful to the firm - buyer A's resale price is increasing in the transfer fee, so imposing one can mitigate resale market competition.

Figure 3 displays the firm's earning with several mechanisms when the firm has two units to sell. Two versions of reselling are evaluated: the firm continues with a zero transfer fee, and the second the firm uses the optimal transfer fee. While the advantage of reselling without a transfer fee is reduced relative to overbooking, it generally remains the preferred mechanism for the firm. The preference for reselling is stronger when the firm imposes the optimal transfer fee to mitigate some of the period 2 competition it may face in the reselling market.

Reselling survives competition for two reasons. First, although the firm and buyer A compete in period 2, they are not on equal footing. The firm has no use for the unit, whereas buyer A does earn some value if buyer A retains the unit. Thus, buyer A is not as price aggressive as the firm. In many situations buyer A's optimal price is greater than the firm's optimal price as the unique seller, thereby allowing the firm to choose its preferred price disregarding the presence of buyer A. Second, the firm can use a transfer fee to

Figure 3. Firm revenue when the firm has two units to sell. Buyers have values which are uniformly distributed $[0, 1]$. Two reselling mechanisms are evaluated, one without a transfer fee, $t = 0$, and the other with the firm's optimal transfer fee, t^*.



mitigate whatever price competition does occur with buyer A. If there are actual transaction costs in the reselling market, then those costs may serve an analogous purpose, reducing the need for the firm to impose a transfer fee.

6 Conclusion

We study how a firm should price it's limited perishable capacity over time. We find it can be optimal for a firm to allow consumers to buy in advance and to resell to other consumers, as in ticket exchanges that are now common in many markets. Enabling an efficient resale market can also benefit consumers, and can serve the secondary goal to eliminate the participation of speculators. We find that a resale market can be desirable for the firm even if it creates the possibility of direct competition between the firm and its customers because consumers are not price aggressive - unlike the firm, consumers value the good to some extent, so are less motivated to sell.

Reselling is not the only recourse method available to the firm. The next best is overbooking - the firm makes an initial sale and then attempts to find a buyer willing to pay even more. The refund mechanism is the least attractive, but still useful, of the recourse options available to the firm.

With any selling mechanism, the goal for the firm is to create value and then extract some of that value. Value is created in two ways: (i) transfer the capacity to a buyer (the firm has zero value for the capacity at the end of the horizon) and (ii) conditional on a transfer, ensure the capacity is used by the buyer with the highest value. Advance selling is better than spot selling at ensuring the capacity is transferred to some buyer, but it is less effective at placing the capacity with the buyer with the highest value. Thus, despite previous

research that extols the virtues of advance selling, advance selling (without recourse) is preferred over spot selling only when demand is somewhat limited. Recourse strategies address the weakness of advance selling (proper allocation). But as already emphasized, there is significant variation in the performance across recourse strategies. In short, the most effective strategy puts the agent with the best information in charge of allocation. Reselling is best because consumers know their value, and only make a trade if it makes sense. Refunds is the laggard because the firm's obligation to accept a return leaves it vulnerable to the possibility that it will not find a seller in the spot market, thereby wasting the capacity that otherwise could have generated some value.

Clearly, recourse strategies can only be effective if actual transaction costs to implement them are not excessive. In markets with high transaction costs, recourse strategies might not be desirable. However, the use of information technology generally reduces transaction costs, making these recourse strategies potentially feasible and even highly profitable. This is consistent with the observation that instead of trying to prevent resale, many sellers of perishable capacity (e.g., sports teams) now actively encourage reselling among their consumers.

References

Authors. 2020. *Selling Capacity Over Time when the Firm is Uncertain of what Customers Know.* Working Paper.

Aviv, Yossi, & Pazgal, Amit. 2008. Optimal Pricing of Seasonal Products in the Presence of Forward-Looking Consumers. *Manufacturing & Service Operations Management*, **10**(3), 339–359.

Biyalogorsky, Eya, Gerstner, Eitan, Weiss, Dan, & Xie, Jinhong. 2005. The economics of service upgrades. *Journal of Service Research*, **7**(3), 234–244.

Biyalogorsky, Eyal, Carmon, Ziv, Fruchter, Gila E., & Gerstner, Eitan. 1999. Research Note: Overselling with Opportunistic Cancellations. *Marketing Science*, **18**(4), 605–610.

Cachon, Gérard P., & Feldman, Pnina. 2011. Pricing services subject to congestion: charge per-use fees or sell subscriptions? *Manufacturing & Service Operations Management*, **13**(2), 244–260.

Cachon, Gérard P., & Feldman, Pnina. 2017. Is advance selling desirable with competition? *Marketing Science*, **36**(2), 195–213.

Cachon, Gérard P., & Swinney, Robert. 2009. Purchasing, Pricing, and Quick Response in the Presence of Strategic Consumers. *Management Science*, **55**(3), 497–511.

Chu, Leon Yang, & Zhang, Hao. 2011. Optimal preorder strategy with endogenous information control. *Management Science*, **57**, 1055–1077.

Courty, Pascal. 2003. Some economics of ticket resale. *The Journal of Economic Perspectives*, **17**(2), 85–97.

Cui, Yao, Duenyas, Izak, & Şahin, Özge. 2014. Should Event Organizers Prevent Resale of Tickets? *Management Science*, **60**(9), 2160–2179.

Dana, James. 1998. Advance-purchase discounts and price discrimination in competitive markets. *Journal of Political Economy*, **106**(2), 395–422.

DeGraba, Patrick. 1995. Buying frenzies and seller-induced excess demand. *The RAND Journal of Economics*, **26**(2), 331–342.

Feldman, Pnina, Papanastasiou, Yiangos, & Segev, Ella. 2019. Social Learning and the Design of New Experience Goods. *Management Science*, **65**(4), 1502–1519.

Gale, Ian L., & Holmes, Thomas J. 1993. Advance-purchase discounts and monopoly allocation of capacity. *American Economic Review*, **83**(1), 135–146.

Gallego, Guillermo, & Sahin, Ozge. 2010. Revenue management with partially refundable fares. *Operations Research*, **58**(4), 817–833.

Gallego, Guillermo, Kou, Steve G., & Phillips, Robert. 2008. Revenue Management of Callable Products. *Management Science*, **54**(3), 550–564.

Guo, Liang. 2009. Service cancellation and competitive refund policy. *Marketing Science*, **28**(5), 901–917.

Huang, Tingliang, Yin, Zhe, & Chen, Ying-Ju. 2017. Managing Posterior Price Matching: The Role of Customer Boundedly Rational Expectations. *Manufacturing & Service Operations Management*, 337–507.

Kahneman, Daniel, Knetsch, Jack, & Thaler, Richard. 1990. Experimental test of the endowment effect and the coase theorem. *Journal of Political Economy*, **98**(6), 1325–1348.

Karaesmen, Itir, & Van Ryzin, Garrett. 2004. Overbooking with substitutable inventory classes. *Operations Research*, **52**(1), 83–104.

Lai, Guoming, Debo, Laurens G., & Sycara, Katia. 2010. Buy Now and Match Later: Impact of Posterior Price Matching on Profit with Strategic Consumers. *Manufacturing & Service Operations Management*, **12**(1), 33–55.

Lariviere, Martin. 2006. A note probability distributions with increasing generalized failure rates. *Operations Research*, **54**(3), 602–604.

Li, C., & Zhang, F. 2013. Advance demand information, price discrimination, and preorder strategies. *Manufacturing & Service Operations Management*, **15**(1), 57–71.

Liu, Qian, & van Ryzin, Garrett J. 2008. Strategic capacity rationing to induce early purchases. *Management Science*, **54**(6), 1115–1131.

Michael Lewis, Yanwen Wang, Chunhua Wu. 2019. Season Ticket Buyer Value and Secondary Market Options. *Marketing Science - article in advance.*

Moe, W. W., & Fader, P. S. 2002. Using advance purchase orders to forecast new product sales. *Marketing Science*, **21**(3), 347–364.

Myerson, Roger B. 1981. Optimal auction design. *Mathematics of Operations Research*, **6**(1), 58–73.

Papanastasiou, Yiangos, & Savva, Nicos. 2017. Dynamic pricing in the presence of social learning and strategic consumers. *Management Science*, **63**(4), 919–939.

Prasad, Ashutosh, Stecke, Kathryn E., & Zhao, Xuying. 2011. Advance Selling by a Newsvendor Retailer. *Production and Operations Management*, **20**(1), 129–142.

Roth, A. 2007. Repugnance as a constraint on markets. *Journal of Economic Perspectives*, **21**, 37–58.

Su, Xuanming. 2010. Optimal Pricing with Speculators and Strategic Consumers. *Management Science*, **56**(1), 25–40.

Technavio. 2015. *Global Secondary Tickets Market 2016-2020.* Tech. rept. https://www.technavio.com/report/global-media-and-entertainment-services-secondary-tickets-market.

Weatherford, Lawrence R., & Bodily, Samuel. 1992. A taxonomy and research overview of perishable-asset revenue management: yield management, overbooking, and pricing. *Operations Research*, **40**(5), 831–844.

Xie, J., & Shugan, Steve. 2001. Electronic tickets, smart cards, and online prepayments: When and how to advance sell. *Marketing Science*, **20**(3), 219–243.

Xie, Jinhong, & Gerstner, Eitan. 2007. Service Escape: Profiting from Customer Cancellations. *Marketing Science*, **26**(1), 18–30.

Yang, Luyi, Debo, Laurens, & Gupta, Varun. 2017. Trading time in a congested environment. *Management Science*, **63**(7), 2377–2395.

A Proofs

Proof of Lemma 1. The transaction fee has no impact on W_2. Define

$$z(t) = U_2(t) + T(t) = \int_0^{1-t} \left[(1 - G(p_2(v,t)))\, p_2(v,t) + vG(p_2(v,t))\right] f(v)dv + \int_{1-t}^{1} vf(v)dv$$

Given that

$$\frac{\partial z(t)}{\partial t} = 0$$

the impact of t on the firm is through its influence on the buyer's period 2 expected value before the transaction fee through the period 2 price, $p_2(v,t)$. But those earnings are strictly decreasing in the transaction fee because the fee distorts the buyer's pricing (i.e., the buyer's optimal price with the transaction fee does not maximize its earnings without the transaction fee). Hence, the firm maximizes its earnings with a zero transaction fee.

□

Proof of Lemma 2. Let $u_2(p_2, v_A)$ be buyer A's period 2 utility from selling in the resale market to the B buyers at price p_2,

$$u_2(p_2, v_A) = (1 - G(p_2))p_2 + G(p_2)\, v_A$$

Let $p_2(v_A)$ be buyer A's optimal resale price,

$$p_2(v_A) = \arg\max_{p_2} u_2(p_2, v_A)$$

Let $\hat{W}_2$ be buyer A's expected utility from waiting to purchase if buyer A is able to purchase in period 2 ahead of all B buyers,

$$\hat{W}_2 = E\left[\left(v_A - p_2^+\right)^+\right]$$

Buyer A cannot do better in period 2 than being first to purchase, so $W_2 \leq \hat{W}_2$. With reselling the firm charges buyer A in period 1

$$p_1 = E\left[u_2\left(p_2(v_A), v_A\right)\right] - W_2 \geq E\left[u_2\left(p_2(v_A), v_A\right)\right] - \hat{W}_2$$

Given $W_2 \leq \hat{W}_2$,

$$p_1 \geq E\left[u_2\left(p_2(v_A), v_A\right)\right] - \hat{W}_2$$

Suppose buyer A were to use the sub-optimal pricing strategy, $\hat{p}_2(v_A) = \max\left\{v_A, p_2^+\right\}$. It follows that

$$p_1 > E\left[u_2\left(\hat{p}_2(v_A), v_A\right)\right] - \hat{W}_2 > \int_0^{p_2^+} u_2\left(p_2^+, 0\right) f(v)dv + \int_{p_2^+}^{1} p_2^+ f(v)dv = u_s$$

where u_s is the speculator's expected earnings from selling to the set of buyers: if buyer A's value is less than p_2^+, then the speculator tries to sell to buyers B. Otherwise, the speculator will surely get p_2^+, because at the very least buyer A is willing to purchase. Thus, the speculator cannot earn a positive profit by buying from the firm, i.e., $u_s - p_1 < 0$.

□

Proof of Theorem 1. Immediately from the f.o.c

$$\frac{d\Pi_c(t_c)}{dt_c} = f(t_c)(-t_c + \pi_2^*)$$

we obtain $t_c^* = \pi_2^*$.

□

Proof of Theorem 2. Differentiation of the firm's revenue function yields

$$\begin{array}{rcl} \frac{d\Pi_o}{dt_o} & = & \frac{\partial \Pi_o}{\partial p_2}\frac{\partial p_2}{\partial t_o} + \frac{\partial \Pi_o}{\partial t_o} \\ & = & (\mu_A - t_o)\, g(p_2(t_o))\, \frac{\partial p_2(t_o)}{\partial t_o} \end{array}$$

Given that $\partial p_2(t_o)/\partial t_o > 0$, the unique optimal overbooking contract has $t_o = \mu_A$.

□

Proof of Theorem 3. Three relationship need to be established.

- $\Pi_s(p_s^*) < \Pi_c(t_c^*)$. The spot selling mechanism is a special case of a refund mechanism: a refund contract with $p_c = t_c = p_s^*$ yields the same revenue for the firm as spot selling. With the optimal refund mechanism $t_c^* < p_c^*$, where p_c^* is the optimal period 1 price. Hence, the optimal refund mechanism generates more revenue for the firm than spot selling.

- $\Pi_c(t_c^*) < \Pi_o(t_o^*)$. Define a mechanism that combines refunds and overbooking: the firm announces two transfers, t_c and t_o, $t_c < t_o$; if $v_A \leq t_c$ (as with refunds), buyer A returns the unit to the firm who is responsible for trying to sell it in period 2; if $t_c < v_A$ (as with overbooking), buyer A retains the unit unless the firm can sell it in period 2, which would trigger a transfer of t_o to buyer A. In period 1 buyer A's utility from the combination mechanism is

  $$U_{co}(t_c, t_o) = t_c F(t_c) + (1 - F(t_c)) \left\{ G(p_2(t_o)) \left(\frac{1}{(1 - F(t_c))} \int_{t_c}^{1} v f(v)\, dv \right) + (1 - G(p_2(t_o)))\, t_o \right\}$$

  The buyer could always wait to period 2 to purchase and earn utility W_2. Hence, the most the firm can charge in period 1 is

  $$p_1(t_c, t_o) = U_{co}(t_c, t_o) - W_2$$

  The firm's revenue from the combination mechanism is

  $$\Pi_{co}(t_c, t_o) = p_1(t_c, t_o) + F(t_c)(\pi_2(p_2(0), 0) - t_c) + (1 - F(t_c))\, \pi_2(p_2(t_o), t_o)$$

  Differentiation reveals that $t_c = 0$ is optimal for the firm:

  $$\begin{array}{rcl} \frac{d\Pi_{co}(t_c, t_o(t_c))}{dt_c} & = & \frac{\partial \Pi_{co}(t_c, t_o(t_c))}{\partial t_c} + \frac{\partial \Pi_{co}(t_c, t_o(t_c))}{\partial t_o}\frac{\partial t_o(t_c)}{\partial t_c} \\ & = & -(t_o - t_c) f(t_c)(1 - G(p_2(t_o))) - f(t_c)\, t_c < 0 \end{array}$$

- $\Pi_o(t_o^*) < \Pi_r(0)$. Let

  $$\pi_2(v) = \max_{p_2} p_2(1 - G(p_2)) + vG(p_2)$$

It follows that

$$\Pi_o\left(t_o^*\right) = \pi_2\left(\mu_A\right) - W_2$$

and

$$\Pi_r\left(0\right) = \int_0^1 \pi_2\left(v\right) f\left(v\right) dv - W_2$$

In other words, with overbooking the good is priced in period 2 as if its value is μ_A whereas with reselling the good is priced for its correct value, v. Given that $\pi_2\left(v\right)$ is strictly convex, from Jensen's inequality, $\Pi_o\left(t_o^*\right) < \Pi_r\left(0\right)$.

□

Proof of Theorem 4. Theorem 3 establishes that selling to buyer A in period 1 is better than selling to the full set of buyers in period 2. It remains to establish that reselling is the firm's optimal mechanism for selling to buyer A in period 1. With any mechanism, buyer A's least amount of surplus is W_2 because buyer A always has the option to reject an offer in period 1. Hence, a mechanism is optimal if it maximizes the combined surplus of the firm and buyer A, while leaving buyer A with only W_2 surplus. Surplus in period 1 is generated if the unit is transferred to buyer A and then if the unit can be transferred in period 2 to a B buyer with higher value. Assuming buyer A and the firm have access to the same mechanism for selling the unit to the B buyers in period 2, the total surplus generated is greatest if buyer A implements the period 2 selling mechanism. For example, assuming they both can use a posted price mechanism (with the same implementation costs), then buyer A posting a price earns more value than the firm posting a price. If the second period selling mechanism were a second-price auction with a reserve (which would maximize the revenue generated in period 2), then again, buyer A generates more total value than the firm. Thus, reselling is optimal because it ensures buyer A performs the selling in period 2 and buyer A only earns surplus W_2.

□

B Reselling competition

The firm has two units to sell and three options for selling: spot sell the two units in period 2 to the full set of buyers (A plus Bs); offer a reselling contract to buyer A; and offer an overbooking contract to buyer A. Let $G_2(p)$ and $g_2\left(p\right)$ be the distribution and density function of the 2nd highest value among the B buyers.

$$G_2\left(p\right) = \begin{cases} nG\left(p\right)^{n-1} - \left(n-1\right)G\left(p\right)^n & 1 \leq n \\ 1 & n < 1 \end{cases}$$

Naturally,

$$G\left(p\right) < G_2\left(p\right)$$

Let $\hat{G}_2\left(p\right)$ be the distribution function of the second highest value among the B buyers ahead of buyer A in the purchase queue in period 2 if buyer A waits (or is forced to wait) to period 2 to attempt a purchase.

B.1 Spot selling

If the firm sells the two units in period 2 with a posted price, then the firm's revenue is

$$\pi_2\left(p_2\right) = \left(2 - G^+\left(p_2\right) - G_2^+\left(p_2\right)\right)p_2$$

Assume there is a unique optimal price, p_2^*. If buyer A anticipates the period 2 price will be p_2^*, then buyer A's expected utility at the end of period 1 is

$$W_2 = \hat{G}_2(p_2) E\left[(V_A - p_2)^+\right]$$

B.2 Reselling

Reselling creates the possibility that buyer A and the firm compete to sell their units in period 2. In that case, buyer A posts a price and then the firm posts a price. Buyer A pays a transfer fee t_r to the firm if it makes a sale. The seller with the lowest price has the opportunity to sell to the B buyer with the highest value, and the other seller has the opportunity to purchase from the B buyer with the second highest value. If the firm and buyer A post the same price, then the firm is considered to have the lower price (i.e., the firm has the ability to slightly undercut buyer A).
If the firm has the lowest price, then its profit in period 2 from selling its unit is

$$\pi_2(p_2) = (1 - G(p_2)) p_2$$

Let

$$p_L^* = \arg\max \pi_2(p_2)$$

be the firm's optimal price when it is assumed to be the seller with the lowest price.
Buyer A's utility from selling, assuming it is the seller with the higher price, is

$$u_H(p_2, v, t_r) = vG_2(p_2) + (1 - G_2(p_2))(p_2 - t_r)$$

Let $p_H(v_A, t_r)$ be buyer A's optimal price when it is the seller with the highest price and value v_A for the unit,

$$p_H(v, t_t) = \arg\max_{p_2} u_H(p_2, v, t_r)$$

Define $\overline{v}$ such that $p_H(\overline{v}, t_r) = p_L^*$.

Theorem 5. *The equilibrium prices in period two are given in the following table*

v_A	Firm	Buyer A
$v_A \leq \overline{v}$	$p_H(v_A, t_r)$	$p_H(v_A, t_r)$
$\overline{v} < v_A$	p_L^*	$p_H(v_A, t_r)$

Proof. If buyer A's value is sufficiently low, $v_A \leq \overline{v}$, then A's optimal price as the "second seller" undercuts the firm's optimal price as the "first seller". The firm's optimal price as the second seller is lower than A's price as a second seller, so the firm has no incentive to lower its price below $p_H(v_A, t_r)$ (because a lower price makes it the first seller, which has an optimal price greater than $p_H(v_A, t_r)$). Buyer A has no incentive to lower its price below $p_H(v_A, t_r)$ because then it would become the first seller (which has an optimal price higher than $p_H(v_A, t)$), nor does it want to raise its price because doing so ensures buyer A remains the second seller and as such, the buyer prefers $p_H(v_A, t_r)$ over a higher price. If buyer A's value is sufficiently high, $\overline{v} < v$, then each seller is selecting its optimal price given its role (first or second seller).□

Buyer A is always the second seller in equilibrium in period 2. Let $U_H(t_r)$ be buyer A's expected utility in period 1 from owning a unit

$$U_H(t_r) = \int_0^{1-t_r} u_H(p_H(v, t_r), v) f(v) dv + \int_{1-t_r}^{1} v f(v) dv$$

Buyer A is willing to pay in period 1 up to p_1 for the unit,

$$p_1 = U_H(t_r) - W_2$$

□

The firm in period 1 earns

$$\Pi_1(t_r) = p_1 + \int_0^{\overline{v}} \pi_2(p_H(v,t_r)) f(v)dv + (1 - F(\overline{v}))\pi_2(p_L^*) + t_r \int_0^{1-t_r} (1 - G_2(p_H(v,t_r))) f(v)dv$$

Without competition the firm minimizes transaction fees. With competition there are two counteracting effects associated with the transaction fee. To illustrate them, note that

$$\begin{array}{rcl} \frac{d\Pi_1(t_r)}{dt_r} & = & \frac{\partial \Pi_1(t_r)}{\partial t_r} + \frac{\partial \Pi_1(t_r)}{\partial p_H}\frac{\partial p_H(v,t_r)}{\partial t_r} + \frac{\partial \Pi_1(t_r)}{\partial \overline{v}}\frac{\partial \overline{v}}{\partial t_r} \\ & = & -t_r \int_0^{1-t_r} \left\{ g_2(p_H(v,t_r)) \frac{\partial p_H(v,t_r)}{\partial t_r} \right\} f(v)dv + \int_0^{\overline{v}} \frac{d\pi_2(p_H(v,t_r))}{dp_H}\frac{\partial p_H(v,t_r)}{\partial t_r} f(v)dv \end{array}$$

and $0 < \partial p_H(v,t_r)/\partial t_r$. Increasing the transaction fee decreases the firm's profit because it distorts buyer A's period 2 pricing decision (the first term) but increases the firm's profit because it mitigates period 2 competition (the second term).

If the transaction fee is sufficiently high, the firm's optimal price is always lower than buyer A's optimal price. Let $\bar{t}_r$ be that unique transaction fee:

$$\bar{t}_r = p_L^* - \frac{1 - G_2(p_L^*)}{g_2(p_L^*)}$$

B.3 Overbooking

If overbooking is done, then the firm's profit in period 2 is

$$\pi_2(p_2, t_o) = (1 - G(p_2))p_2 + (1 - G_2(p_2))(p_2 - t_o)$$

Let $p_2(t_o)$ be the firm's optimal period 2 price (and assume there is a unique optimal price). In period 1 buyer A's utility from the contract is

$$U_o(t_o) = \mu_A G_2(p_2(t_o)) + (1 - G_2(p_2(t_o)))t_o$$

The buyer could always wait to period 2 to purchase and earn utility W_2. Hence, the most the firm can charge in period 1 is

$$p_1(t_o) = U_o(t_o) - W_2$$

The firm's utility in period 1 is

$$\Pi_o(t_o) = p_1(t_o) + \pi_2(p_2(t_o), t_o)$$

Given $\partial p_2/\partial t_o > 0$ and

$$\begin{array}{rcl} \frac{d\Pi_o}{dt_o} & = & \frac{\partial \Pi_o}{\partial p_2}\frac{\partial p_2}{\partial t_o} + \frac{\partial \Pi_o}{\partial t_o} \\ & = & (\mu_A - t_o) g_2(p_2(t_o)) \frac{\partial p_2}{\partial t_o} \end{array}$$

the optimal unique buyback is $t_o^* = \mu_A$. It follows that buyer A's utility is $U_o\left(t_o^*\right)$ and the firm's revenue is

$$\Pi_o\left(t_o^*\right) = p_1\left(t_o^*\right) + \pi_2\left(p_2\left(t_o^*\right), t_o^*\right)$$